University of Saskatchewan IEEE Student Branch

ELECTRICAL ENGINEERING 2nd YEAR EXAM FILE

(Term 1)

2003 Edition

Includes:

EE 201
EE 216
EE 221
EE 271
CMPT 116
MATH 223

Prepared for you by the IEEE

Additional exams available on class web sites and at http://ieee.usask.ca

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Nar _____ **Student Number:**

de 1)

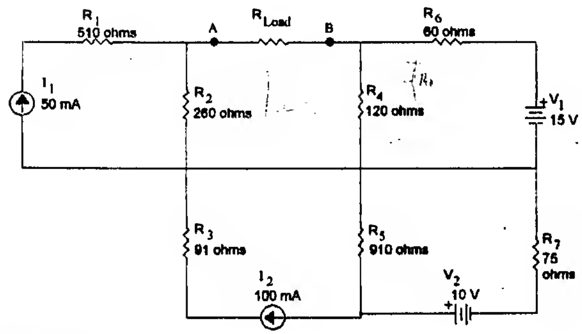


Figure 1

a) Determine and sketch the Thevenin's equivalent circuit for the output terminals A and B.

4.77V

A B

$\boxed{-4.77V}$ ← this should be pos

1

EE221 Midterm Exam

Name: ____________

Studen ____________

Welcome to the EE221 Midterm. This is a closed book examination. You may have two sheets of notes. You may also use a calculator.

Answer only 4 of the 6 problems!

Each problem is worth 25 points. Show your work; credit will be given only if the steps leading to the answer are clearly shown. You might want to indicate voltages and currents on the schematics if appropriate. Partial credit will be given for partially correct answers but only if correct intermediate steps are shown.

NOTE: Use the second approximation for all diodes!

Also assume that all clippers are stiff and clampers have a long time constant compared to the input frequency

Grading

1. 21/25
2. —
3. —
4. 5/25
5. 22/25
6. 16.5/25

Total ~~64.5~~ 65

"Of all the things that I have lost, I miss my mind the most!"

Name:_ _ Student Number:_

2) (Use the second approximation of the diode!)

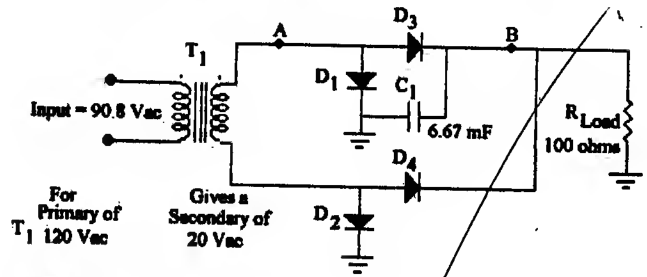


Figure 2

a) What is the peak or maximum voltage across the load?

V_{Load} = ____________

Name:_ _ Student Number:_

Question 1 Continued

b) At what value of load resistance will 600 µA flow through the load?

R_{Load} = 7K652 Ω

$V = IR$

$I = \frac{V}{R} = \frac{V_{Th}}{R_{Th+L}}$

$R_L = \frac{V_{Th}}{I} - R_{Th} = \frac{4.77V}{600\mu A} - 298\Omega$

$R_L = 7652\Omega = 7K652\Omega$

c) At what value of load resistance is maximum power transferred?

R_{Load} = 298 Ω

$R_{Load} = R_{Th}$ of maximum power

d) What would be the minimum load resistance if this Thevenin's equivalent circuit was to be a stiff voltage supply?

$R_{Load(min)}$ = 29K8 Ω

Stiff voltage supply $R_S < 0.01 R_L$

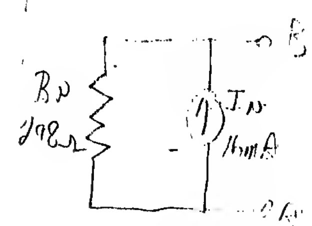
$R_L > 100 R_S$ → $R_L > 29K8\Omega$

$R_L > 100(298\Omega)$

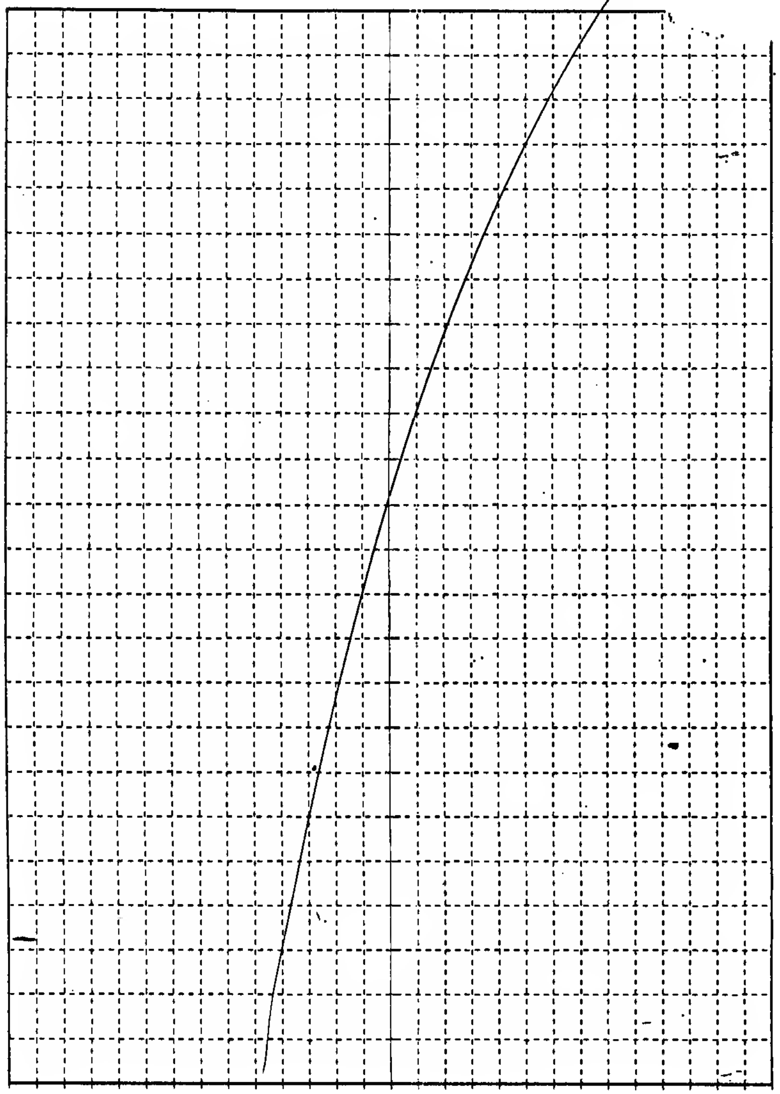
e) What is the Norton's equivalent circuit?
(Provide a circuit sketch as well)

$R_N = R_{Th} = \boxed{298\Omega}$

$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4.77V}{298\Omega} = \boxed{16mA}$



Name:_ _ Student Number:_



Name:_ __ Student Number:

Question 2 Continued

b) What is the peak to peak ripple voltage at the load?

V_{ripple} = ____________

c) What rectifier configuration is this supply?

d) Using the graph on the next page, sketch the waveform at point 'A'. Also on the same graph sketch the resulting waveform at point 'B' both with and without the capacitor in circuit. **(You must indicate which waveform is which along with voltage levels.)**

Please try to be neat! A sloppy diagram may lead to a misinterpretation and lost marks.

Name:_ _ Student Number:_

3) **(Use the second approximation of the diode!)**
Sketch the waveform at each terminal, A, B, C, and D on the supplied graphs.

Please try to be neat! A sloppy diagram may lead to a misinterpretation and lost marks.

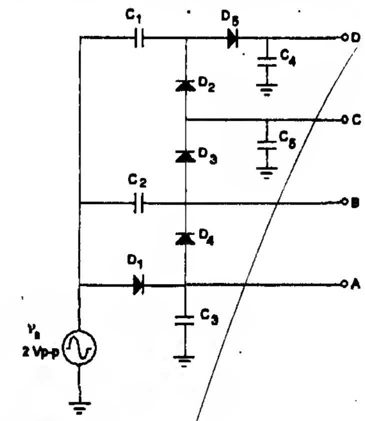
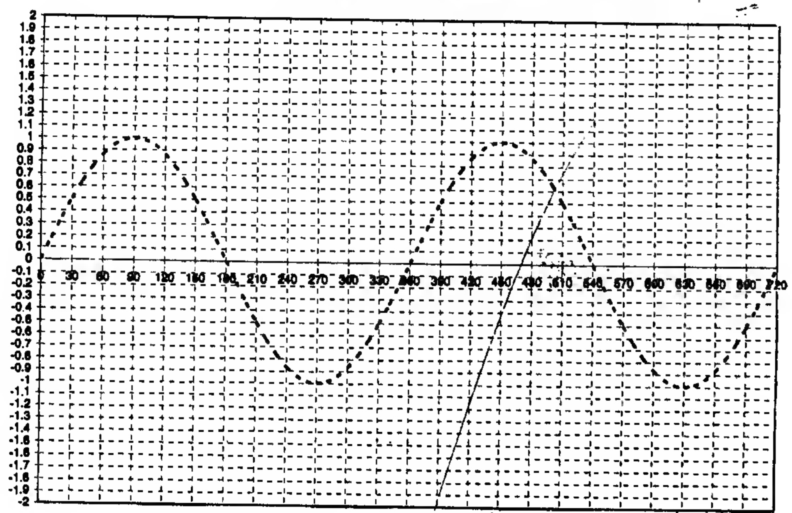


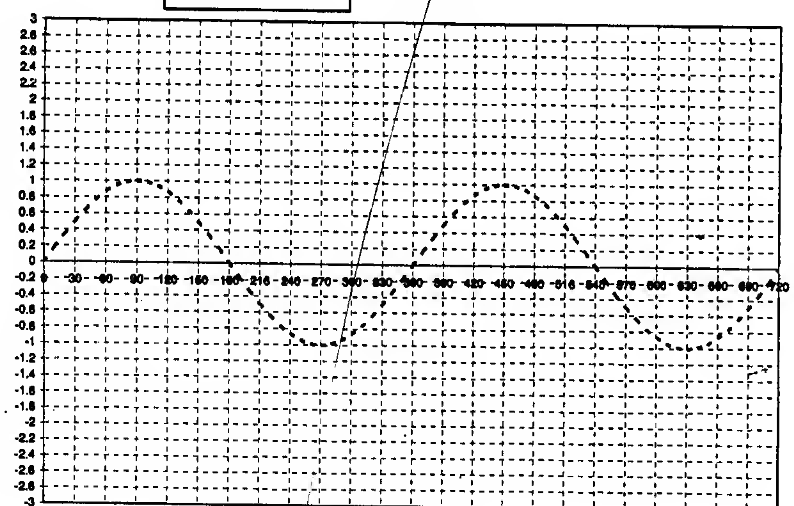
Figure 3

Name ___ Student Number:_

Question 3 Continued



Graph 1 for V_A



Graph 2 for V_B

Name:_ _ **Student Number:**_

4) **(Use the second approximation of the diode!)**
With the given V_{in} sketch V_{out} on the same graph.

Please try to be neat! A sloppy diagram may lead to a misinterpretation and lost marks.

$V_{out} = V_1 + 0.7V$
$V_1 = V_{in} - 0.7V$
$V_{out} = V_{in} - \cancel{0.7 + 0.7V}$
↳ but not for all cases

$V_1 > 3.3V$ D_2 reverse biased
$\therefore V_{out} = 4V$
upper limit

$\therefore D_1$ reverse biased
$V_{out} = V_1 + 0.7V = 1.35V$

+1.35 ↖ lower limit.

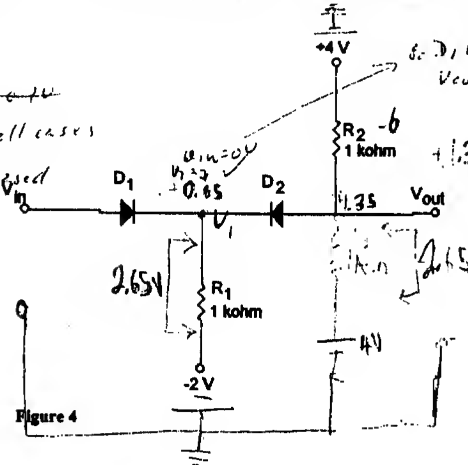
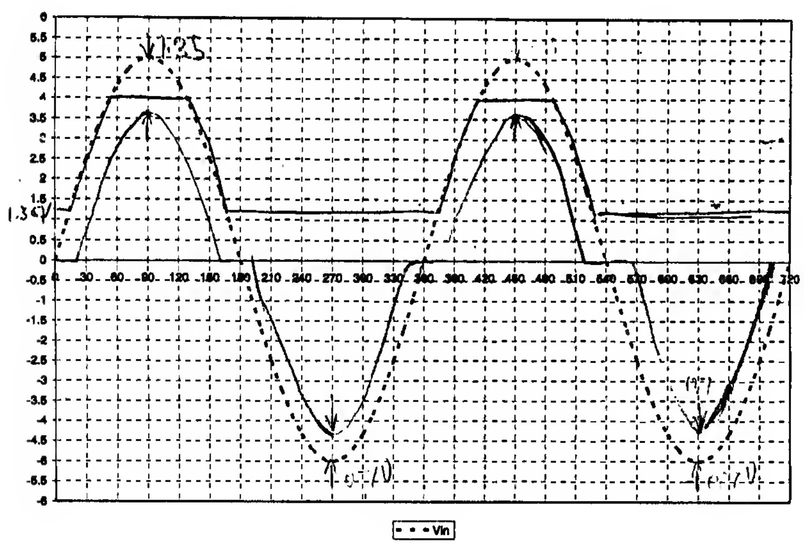


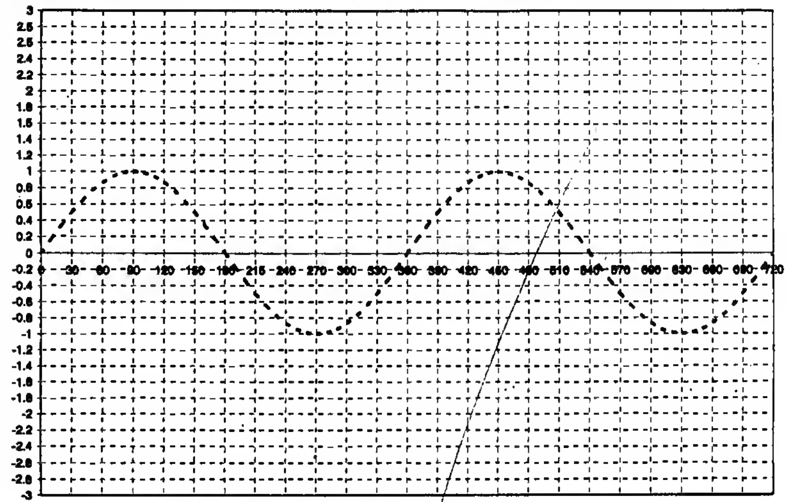
Figure 4

8/25

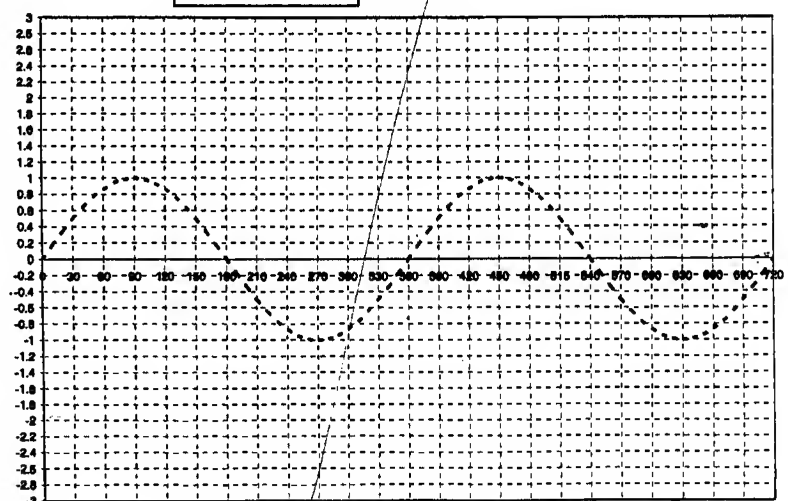


Name:_ _ **Student Number:**_

Question 3 Continued



Graph 3 for V_C



Graph 4 for V_D

Name:_ _ Student Number:_

Question 5 Continued

b) It is determined that a capacitor can be used in order to increase the switching time. On the circuit diagram sketch where this capacitor is to be connected.

6.25

c) If this was a 621 pF capacitor, what would the maximum switching frequency be?

F_{max} = 700 KHz

6.25

$C_{max} = \frac{t_s}{2.3R}$

Since $t_s = \frac{1}{f_{max}}$ $R = R_{C2} // R_{C1}$

$f_{max} = \frac{1}{(C_{max})2.3R} = \frac{1}{(621 \times 10^{-12} F)(2.3)(2k\Omega // 2k\Omega)}$

$\boxed{f_{max} = 700 KHz}$

d) List two advantages the transistor Schmitt Trigger has over a Basic Transistor Switch.

1) With a Schmitt Trigger the circuit will stay on until it drops below LTP and [illegible] until it rises above UTP.

3.25 / 6.25

2) ~~This keeps the input [illegible] with [illegible] the output would be inverted.~~

Name:_ _ Student Number:_

5) (Assume $\beta = 100$, $V_{BE} = 0.7$ V and $V_{CE(sat)} = 0.3$ V)

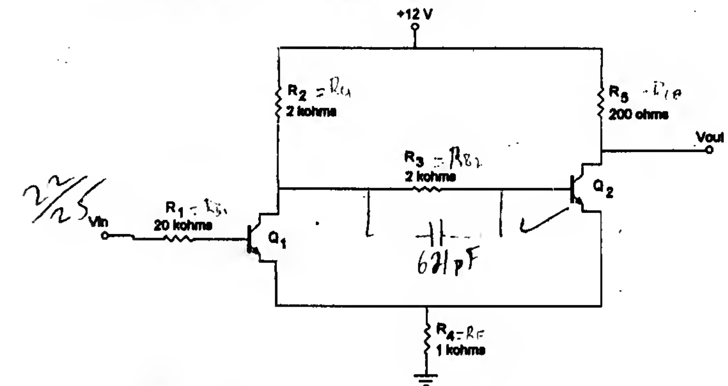


Figure 5

Find the following for the above transistor Schmitt trigger.

a) Determine the:

- Upper Trip Point? UTP = 8V
- Lower Trip Point? LTP = 4V
- Hysteresis Voltage? V_H = 4V

11.25 / 6.25

UTP

$V_{RE} = \frac{R_E}{R_E + R_{C2}}(V_{CC}) = 12V\left[\frac{1k\Omega}{1k\Omega + 500\Omega}\right] = \boxed{8V}$

?

LTP

$V_{RE} = \left(\frac{R_E}{R_E + R_{C1}}\right) V_{CC} = 12V\left[\frac{1k\Omega}{1k\Omega + 2k\Omega}\right] = \boxed{4V}$

V_H

$V_H = UTP - LTP = 8V - 4V = \boxed{4V}$

Name:_ _ Student Number:_

Question 6 Continued
Page for Q point calculations

$V_B = V_{CC}\frac{R_2}{R_1+R_2} = 20V\left(\frac{3k\Omega}{17k\Omega+3k\Omega}\right) = \underline{3V}$

$V_E = 3V - 0.7V = 2.3V$

$I_E = \frac{V_E}{R_{E1}+R_{E2}} = \frac{2.3V}{175\Omega+2k05\Omega} = \underline{1.03mA}$

$I_C = \frac{\beta}{(\beta+1)} I_E = \frac{100}{101}(1.03mA) = \boxed{1.02mA}$ X

$V_C = V_{CC} - I_C R_C = 20V - (1.02mA)(12k\Omega) = \underline{8V}$

$V_{CEQ} = V_C - V_E = 8V - 2.3V = \boxed{5.7V}$ with your I_C

$$I_{C(sat)} = \frac{V_{CC} - \overset{0V}{(0.3V)}}{R_3 + R_4 + R_5} = \frac{20V - 0.3V}{12k\Omega + 175\Omega + 2k05\Omega} = 1.38mA$$

Name:_ _ Student Number:_

6) (Assume $\beta = 100$, $V_{BE} = 0.7$ V, $V_{CE(sat)} = 0.3$ V, and at room temperature (300 °K))

16.5/25

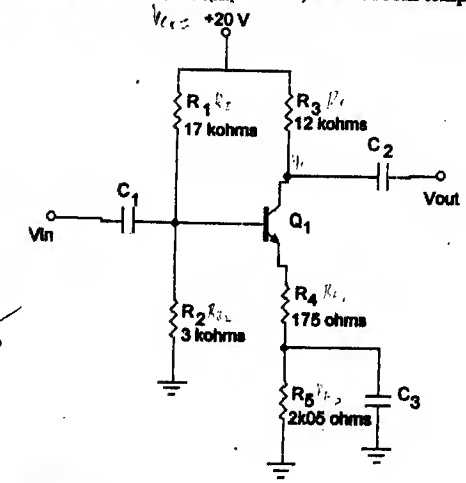
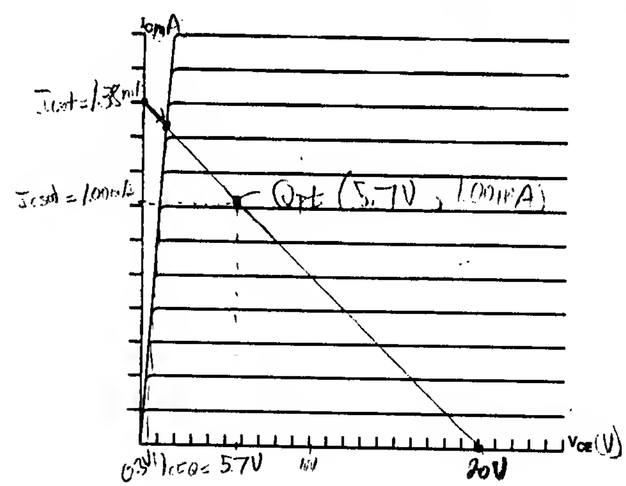


Figure 6

a) Draw the DC load line and determine the Q point.
(Use the supplied transistor curve and label the X and Y axis' values)

I_{CQ} = 1.00mA X V_{CEQ} = 5.7V

3/5

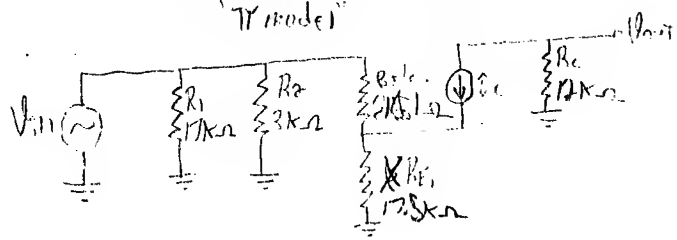


Name:_ _ Student Number:

Question 6 Continued

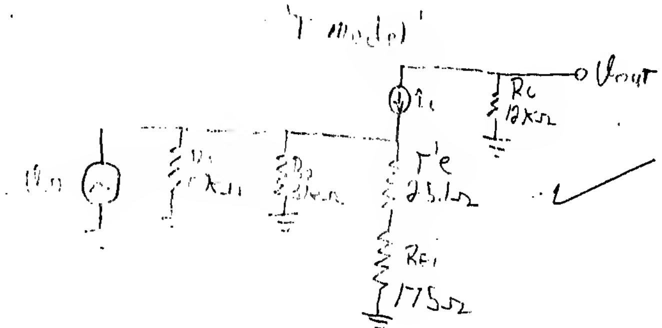
c) Sketch the T and π transistor ac models for this circuit.
(Remember to label all components)

'π model'



4/5

'T model'



Name:_ _ Student Number:

Question 6 Continued

b) Determine the input impedance (Zin), output impedance (Zout) and voltage gain (Av).

Zin = 2K26Ω Zout = 12KΩ Av = -60

4.5/5

$Z_{out} = R_C = 12k\Omega$

$r'_e = \frac{0.025V}{I_E} = \frac{0.025V}{1.03mA} = 25.1\Omega$

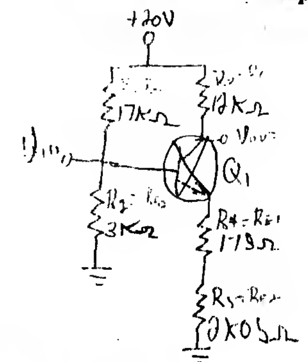
$Z_{in} = R_1 // R_2 // \beta(r'_e + R_{E1}) = 17k\Omega // 3k\Omega // 100(25.1\Omega + 175\Omega) = 2K26\Omega$

$A_V = -\frac{R_C}{r'_e + R_{E1}} = -\frac{12k\Omega}{(25.1\Omega + 175\Omega)} = -60$

Name: **Student Number:**

Question 6 Continued

d) Sketch the DC transistor model for this circuit.
(Remember to label all components)



DC model not circuit.

0/5

e) Is this a stiff voltage divider bias?
(You must show proof to support your answer)

For stiff VDB we need to satisfy $R_1 // R_2 < 0.01\,\beta R_E$

$R_1 // R_2 = 17k\Omega // 3k\Omega = 2K55\Omega$

$R_E = 1(R_{E1} + R_{E2}) = 1(175\Omega + 2K05\Omega) = 2K25\Omega$

∴ Since $R_1 // R_2 < R_E$

$2550\Omega \not< 2250\Omega$

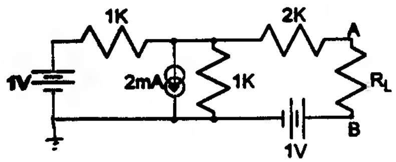
The voltage Divider is not stiff

5

Problem 1

a) Convert the following system of sources and resistors into a Thevenin equivalent circuit. The load resistor R_L is attached between nodes A and B.

b) At what value of R_L is the power dissipated by the load maximized?



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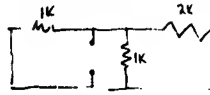
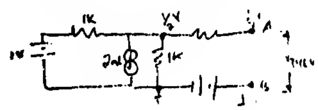
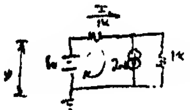
First, Remove Load, "zero" voltages:



$R_{THEV} = 2k\Omega + 1k\Omega \| 1k\Omega$

Now, determine voltage at point AB





$I_1 = 2mA + I_2$ ✓

$1V - I_1(1k\Omega) - V_{cs} = 0$

$V_{cs} = (1k)(I_2)$

put ① into ②

$1V - (2mA + I_2)1k\Omega - V_{cs} = 0$

$1V - (2mA + \frac{V_{cs}}{1k})1k - V_{cs} = 0$

$1V - 2V - V_{cs} - V_{cs} = 0$

$-2V_{cs} = 0$

$V_{THEV} = -0.5V$ x

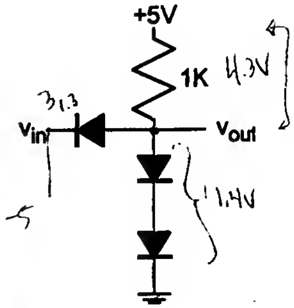
For max power:

R_L should be $= R_{THEV} = 2.5\,k\Omega$ for max power ✓

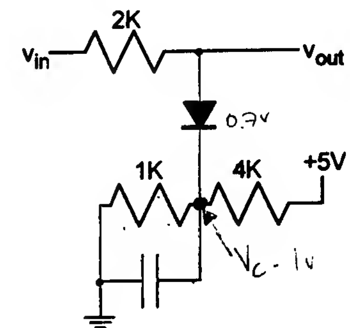
Problem 2

Graph the output voltage v_{out} of the following circuits given the sine wave input voltage v_{in} as shown (draw the curve on the graphs provided). The diodes are silicon, pn junction diodes; use the second model approximation for the diodes. Explain the purpose of the capacitor in part b (the capacitor's value is very large so that it has a small impedance at the frequency of the input signal).

a)



b)



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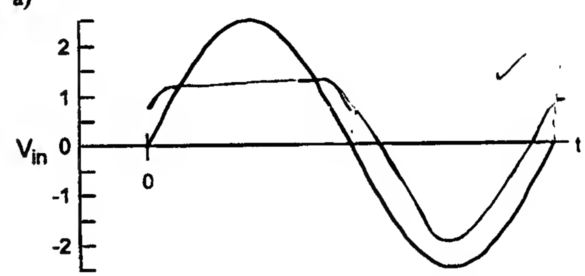
a)

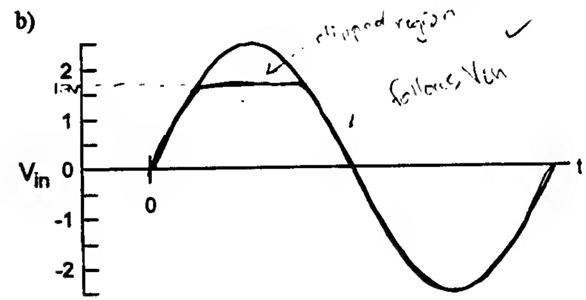


$V_{out} = 0.7V$

$V_{in} = 1.4V$ $V_{out} = 1.4V$

$V_{in} < 0.7V$ $V_{out} = V_{in} + 0.7$

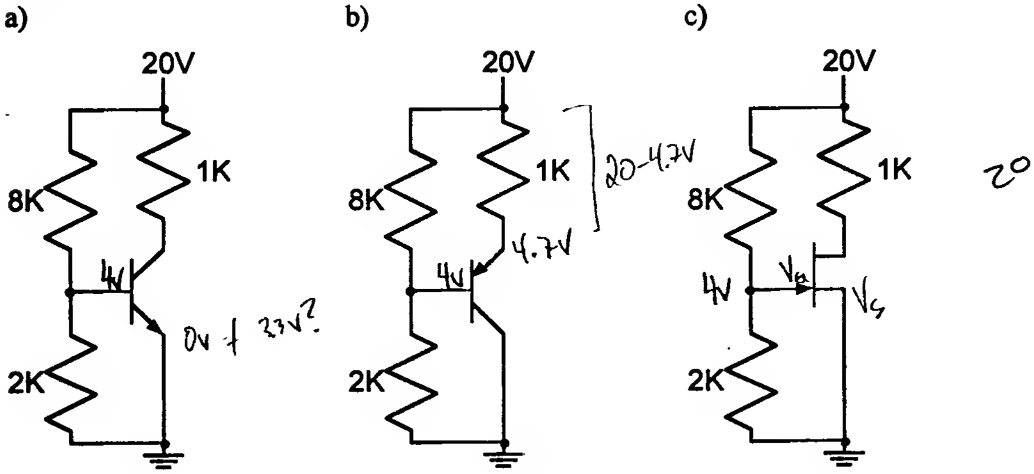
b)



The capacitor allows the signal to bypass the 1K ohm resistor and sets the voltage on Vc equal to 1V. This plus the 0.7V across the diode creates a clipper circuit clipping any voltage over 1.7V.

Problem 3

Determine whether the transistors in the following three circuits are correctly biased. If not, explain what is wrong.



a) Voltage Divider:

$\left(\frac{2K}{10K}\right)(20V) = 4V = V_B$

$V_E = 3.3V$ but without emmitter biasing $3.3V \neq 0V$

∴ Incorrectly biased due to lack of emmitter resistor. ✓

b) Same Voltage Divider:

$V_B = 4V$

∴ Voltage across 1k resistor is 20V − 4.7V

Current can be determined!

Correctly biased! ✓

c) Same Voltage Divider.

$V_G = 4V$

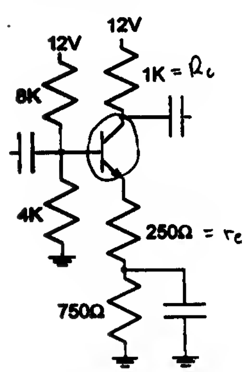
$V_S = 0V$

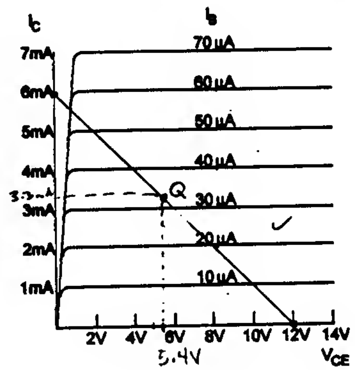
$V_{GS} = 4V$!

∴ Incorrectly biased due to positive V_{GS}. V_{GS} should be negative. The circuit requires a source resistor. (to be correctly biased) ✓

Problem 4

For the following amplifier, draw the dc load line on the collector curves. Determine and indicate the Q point. Calculate the small signal gain of the amplifier and the input and output impedances.





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Dc Analysis: Load Line:

If $V_{CE}=0$ then $I_{max} = \frac{12V}{1k + 250\Omega + 750\Omega} = \frac{12V}{2k} = 6mA$

If $I_c = 0$ the $V_{CE} = 12V$

Dc Analysis at Operation point:

$V_B = \left(\frac{4K}{8K+4K}\right)(12V) = 4V \therefore \boxed{V_E = 3.3V} \therefore I_E = I_C = \frac{3.3V}{1k\Omega} = \boxed{3.3mA}$

$V_{CE} = 12V - (1K)(3.3mA) - 3.3V = \boxed{V_{CE} = 5.4V}$

Signal Gain:

$\frac{25mV}{3.3mA} = 7.576\Omega$

$A = \frac{R_c}{r_e + r_e'} = \frac{1K\Omega}{250\Omega + 7.576\Omega} = 3.88$ $\boxed{A = 3.88}$

Input Impedance: Assume $\beta = 100$

$Z_{in(stage)} = R_1 \| R_2 \| \beta(r_e + r_e') = 4K \| 8K \| 100(250\Omega + 7.576\Omega)$

$= \boxed{2.42 K\Omega}$

Output Impedance

$Z_{out} = R_c \| \left(r_e' + \frac{R_1 \| R_2}{\beta}\right)$ X this formula is for emitter follower.

$= 1K\Omega \| \left(7.576\Omega + \frac{4K \| 8K}{100}\right) = \boxed{33.1\Omega}$

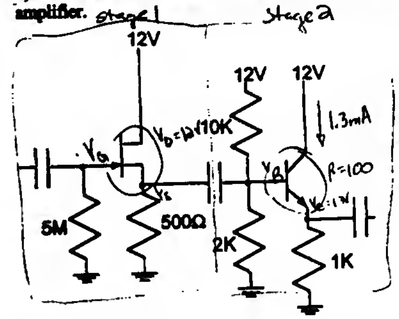
Problem 5

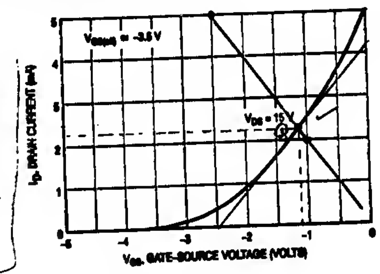
The amplifier below consists of two stages.

a) Determine the bias voltages and currents for each transistor. For the JFET use the transconductance curve provided and indicate the Q point. What type of amplifier is each stage?

b) Determine the input and output impedance of each stage and the overall gain of the amplifier. stage 1 — stage 2

18





Voltage Divider:

$V_B = \left(\frac{2K\Omega}{12K\Omega}\right)(12V) = 2V \therefore \boxed{V_E = 1.3V} \therefore I_E = I_C = 1.3mA$ $r_e' = \frac{25mV}{1.3mA} = 19.2\Omega$

$V_{CE} = 12V - 1.3V$

$\boxed{V_{CE} = 10.7V}$

Find Q point: FET

$\boxed{V_G = 0}$ say $V_S = 1V$ $I_D = \frac{1V}{500\Omega} = 2mA$ @ $V_{GS} = -1V$

say $V_S = 2.5V$ $I_D = \frac{2.5V}{500\Omega} = 5mA$ @ $V_{GS} = -2.5V$

From graph:

$V_{GS} = -1.15V$

$I_D = 2.35 mA$

$V_S = 1.15V$

$V_{DS} = 11.85V$

Slope of graph: $\frac{\Delta I_D}{\Delta V_{GS}} = \frac{4.25 mA}{2.5 V} = 1.7 mS$

Stage 1 = common source JFET Amplifier
stage 2 = emmitter follower amplifier

Input Impedance Stage 2

$Z_{in\,stage} = 2K \| 10K \| \beta(r_e' + R_E)$ $2K \| 10K \| (100)(19.2\Omega) = \boxed{890\Omega}$ X $= Z_{in\,stage\,2}$

$Z_{out\,stage} = 1K \| \left(r_e' + \frac{R_1 \| R_2 \| R_S}{\beta}\right) = 1K \| (19.2\Omega + \frac{2K \| 10K}{100})$ 500Ω

$\boxed{34.62\Omega}$ X $= Z_{out\,stage\,2}$

Input Impedance stage 1 $R_{GS} = \frac{V_{GS}}{I_{GSS}}$ very large

$Z_{in} = \boxed{5 M\Omega}$

$Z_{out} = 500\Omega \| \frac{-V_{GS}}{I_D}$ $\boxed{Z_{out} = 291.6\Omega}$ X

Gain = $A_1 = \frac{g_m r_s}{1 + g_m r_s}$ $A_2 = \frac{r_E}{r_e' + r_E} = \frac{1K}{19.2\Omega + 1K} = 0.981$ ok

$A_1 = (1.7mS)(500\Omega \| 890\Omega)$ ...

$A_{Tot} = A_1 A_2 = \boxed{0.346}$

EE221.3
Midterm 1

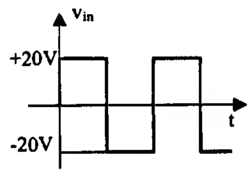
October 19, 2000
Time: 1hr 15min
Open book. Do all 5 questions (4 marks each).
Total mark= 20

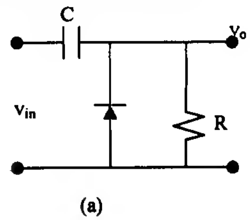
Name: ____________
Student #: ____________

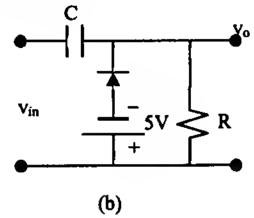
1. **Question 1**

1.5

Sketch v_o of the networks below for the input shown. .The diodes are ideal.



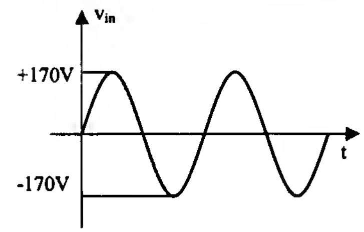


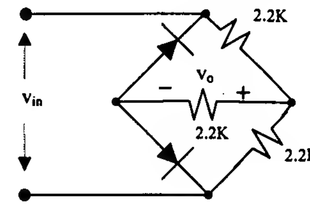


2

2. **Question 2**

Sketch v_o for the network of Figure below and determine the DC voltage available. All the diodes are ideal and all resistors are 2.2K.



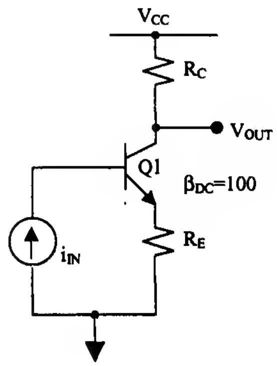


fullwave

3.5

3. **Question 3**

Consider the BJT circuit below with Vcc=15V and $R_C=R_E=5K$. The input source i_{IN} produces a current waveform that begins at zero, rises linearly to a peak of 10μA in 0.5 second, and then falls linearly back to zero at the same rate. Find and sketch the output voltage V_{OUT} versus time if the transistor has a β_{DC} of 100.



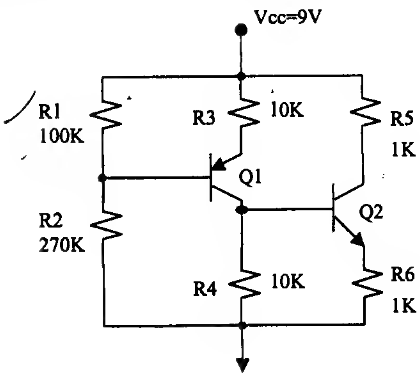
This is V_{RC}, not V_{out}

$V_{out} = V_{CC} - V_{RC}$

3

4. **Question 4**

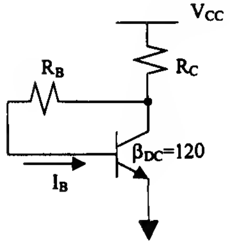
Find all the node voltages (i.e., base, emitter, collector voltage of Q1, Q2) and all resistor currents of the circuit below. Assume very high DC current gain for the transistors.



4

5. **Question 5:**

Prove that the base current of the circuit below is:



$$I_B \approx \frac{V_{CC} - V_{BE}}{R_B + \beta R_C}$$

For V_{CC}=20V, R_C=4.7K, R_B=680K and β_{DC}=120, find I_B, I_C, V_C, V_E, V_{CE}.

Prove ?

2.5

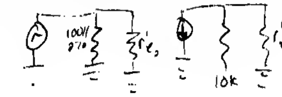
72

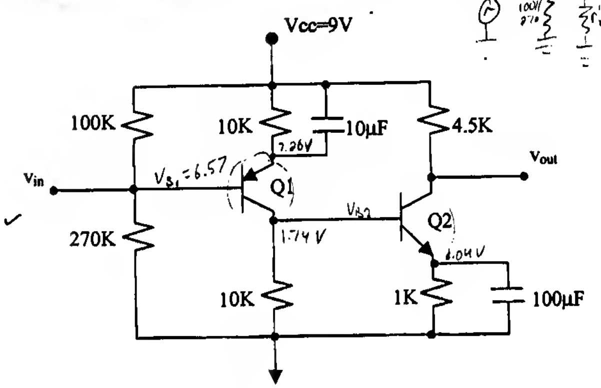
EE221.3
Midterm 2

November 23, 2000
Time: 1hr 15min
Open book, open notes.
Do question 1 and other two (out of questions 2, 3 and 4).
Total mark= 100

Name: ______________
Student #: ______________

1. **Question 1** (50 marks)
   All transistors in the amplifier circuit below have β_{ac}=200 and V_{BE}=0.7V.
   a. Determine midband gain
   b. Find two lower cutoff frequencies due to two bypass capacitors.
   c. Determine the dominant cutoff frequency f_1.
   d. Sketch frequency response of the amplifier and determine the amplifier midband frequency. It has been known that the high cutoff frequency of the amplifier f_2 is 1MHz.
   e. What is the expression of the output waveform, v_{out}, if the waveform applied to the input v_{in} is v_{in}=(0.1mV)sin(2.π.33.t).





f: 53Hz

r'_{e_1} = 143.7Ω

I_{E_1} = 174μA

V_{C_1} = (174μA)(10kΩ)

V_{E_1} = 1.74V

I_{E_2} = 1.04mA

r'_{e_2} = 24.27Ω

V_{B2} = (30.5)(0.1mV) = 3.05 mV ?

Z_{in_2} = 485kΩ   10k

Z_{out_2} = 4.5kΩ

A_2 = 1.079

32

$Z_{in(1)}$ = 100k // 270k // (200)(143.7)

= 7.81 kΩ

20k

$A_1 = \frac{10k // 7.81k}{143.7} = 30.5$

$A = 30.5)(1.079)$

a) $\boxed{A = 32.9}$

$A_1 = \frac{10k}{144\Omega} = 69$

$A_2 = \frac{4.5k}{26\Omega} = 18$

$Z_{out(1)} = 5k = \beta r'_{e_1}$

$A = (A_1)(A_2)(1)(\frac{5}{15}) = (69)(180)(\frac{1}{3}) = 1$

f_i: 33Hz $X_{c_1} = \frac{1}{166\pi(10\mu F)} = 482.3\Omega$

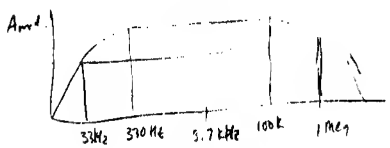
$f = \frac{1}{2\pi(482.3)(10\mu F)} = \quad f_1 \approx 33\,Hz$

b) 33 Hz not in here!!!

$X_{c_2} = \frac{1}{66\pi(100\mu F)} = 48.23\Omega$

$f = \frac{1}{2\pi(48.23)(100\mu F)} = 33Hz$

c) Dominant Frequency Cutoff: 33Hz



$$33.9 = \frac{A_{mid}}{\sqrt{1+\left(\frac{33}{3.70}\right)^2} + \sqrt{1+\left(\frac{3.7k}{1M}\right)^2}}$$

$A_{mid} = 66$

$V_{out} = 3.09\,mV \sin(2\pi 33t)$

(b) $Z_{eq} = 10k \,||\, (r_e' + \frac{R_1 || R_2}{\beta})$
$= 484\Omega$

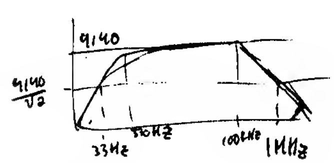
$f_1 = \frac{1}{2\pi(Z_{eq})C} = 33Hz$

$f_1 = \frac{1}{2\pi Z_{eq} C} = \frac{1}{2\pi(70\Omega)(100\mu F)} = 23Hz$

$Z_{eq} = 1k \,||\, (r_e' + \frac{10k}{\beta}) = 70\Omega$

(c) dominant frequency : $f_1 = 33Hz$

(d)



2

(e) $V_{in} = 0.1 V \sin(2\pi 33t)$

$f = f_1$: $A = 0.707(A_{mid}) = 0.707(4140) = 2927$

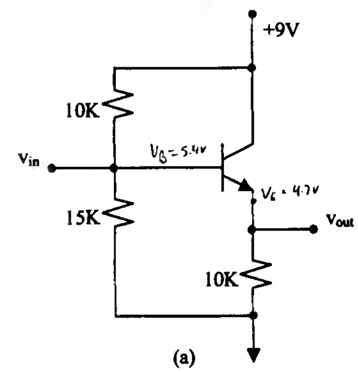
$V_o = 292.7\,mV \sin(2\pi 33t) + 4.5V_{dc}$

2. **Question 2** (25marks)

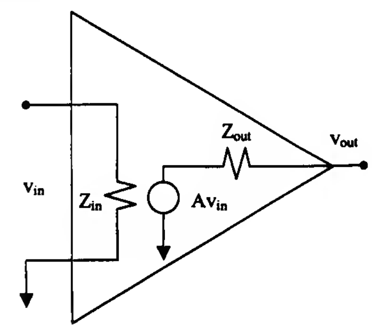
The amplifier in (a) can be represented as (b). The transistor in the amplifier has β_{ac}=200 and V_{BE}=0.7V.

a. What are Z_{in}, A and Z_{out}?

b. What is the output voltage if an input voltage of 200mV is applied to v_{in} and a 10KΩ load is connected from v_{out} to ground?



(a)



(b)

$r_e' = \frac{25mV}{.470mA} = 53.19\Omega$

$\beta r_e' = 10.64k\Omega$

$Z_{in} = 10k \,||\, 15k \,||\, 200(10k + 53.19\Omega)$

$\boxed{Z_{in} = 5.98\,k\Omega}$

$V_{out} = i_e r_e$
$= 19.89\mu A(10k\Omega)$
$= 198\,mV$

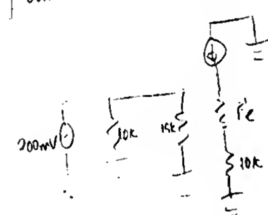
$V_{in} = i_e(r_e + r_e')$

$A = \frac{r_e}{r_e + r_e'} = \frac{10k}{10k + 53.19} = 0.9947$

$Z_{out} = 10k \,||\, (53\,\Omega + \frac{10k || 15k}{200})$

$\boxed{Z_{out} = 82.50\Omega}$

$\boxed{A = 0.9947}$



$V_{out} = i_e(5k\Omega)$

$\boxed{V_{out} = 99.45\,mA}$

???

21

$200mV = i_e(10k + 53.19)$

$i_e = 19.89\mu A$

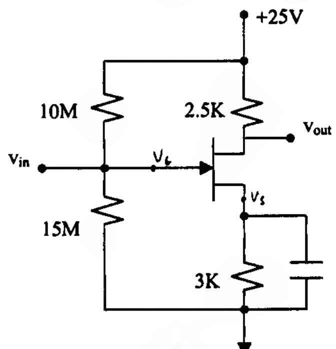
3

3. **Question 3** (25marks)

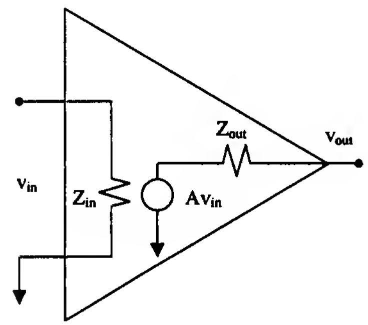
The amplifier in (a) can be represented as (b). The transistor in the amplifier has I_{DSS}=20mA and V_P=+4V.

a. What are Z_{in}, A and Z_{out}?

b. What is the output voltage if a 10KΩ connected from v_{out} to ground and an input voltage of 1mV is applied to v_{in}?



(a)



(b)

$I_{DSS} = 20mA$

$V_P = 4V$

$V_{GS(off)} = -4V$

$V_G = 15V$

$g_{mo} = \frac{40mA}{4V} = 10mS$

$I_D = \frac{V_{DD}}{R_S} = 10mA$

$\frac{I_D}{I_{DSS}} = \frac{1}{2} = \left(1 - \frac{V_{GS}}{V_{GS(off)}}\right)^2$

$V_{GS}^2 - 2V_{GS} + \frac{1}{2} = 0$

$\frac{2 \pm \sqrt{2}}{2} = 0.293$

$g_m = (10mS)(1 - 0.293)$

$g_m = 7.07mS$

$A = 7.07 \times 10^{-3}(2.5k)$

$\boxed{A = 17.68}$

$\boxed{Z_{in} = 6M\Omega}$

$Z_{out} = 2.5k\Omega$

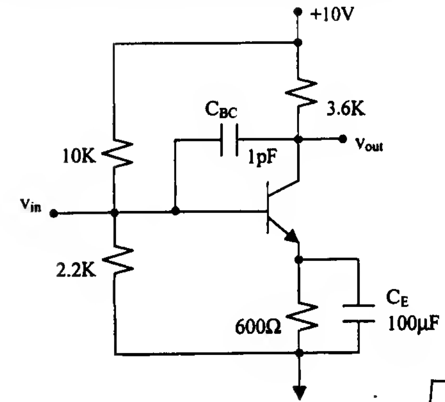
$V_{OUT} = 7.07 \times 10^{-3}(1 \times 10^{-3})(2.5k) = 17.68 mV$

$\boxed{V_{out} = 17.68mV}$

19

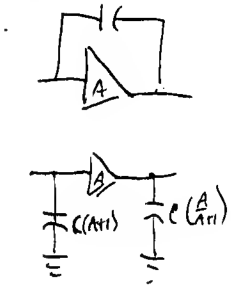
4. **Question 4** (25 marks)

Find all the cutoff frequencies due to C_{BC} and C_E of the amplifier circuit shown below. The transistor has β_{ac}=200 and V_{BE}=0.7V. (Note: 1μF=10^{-6}F, 1pF=10^{-12}F). *Hint: for high cutoff frequencies, convert the feedback capacitor C_{BC} into input and output capacitors.*

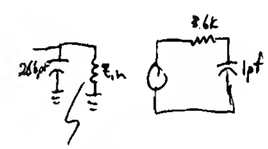


$r_e' = 13.6\Omega$

$A = \frac{R_C}{r_e'} = 265$



$f_1 = \frac{1}{2\pi(1000 \| (r_e' + \frac{10k \| 2.2k}{\beta}))(100\mu F)} = 73Hz$



$2.2k \| 10k \| \beta r_e' = 1.084k\Omega$

$f = \frac{1}{2\pi(1.084k)(266pF)} = 552kHz$

$f = \frac{1}{2\pi(3.6k)(1pF)} = 44MHz$

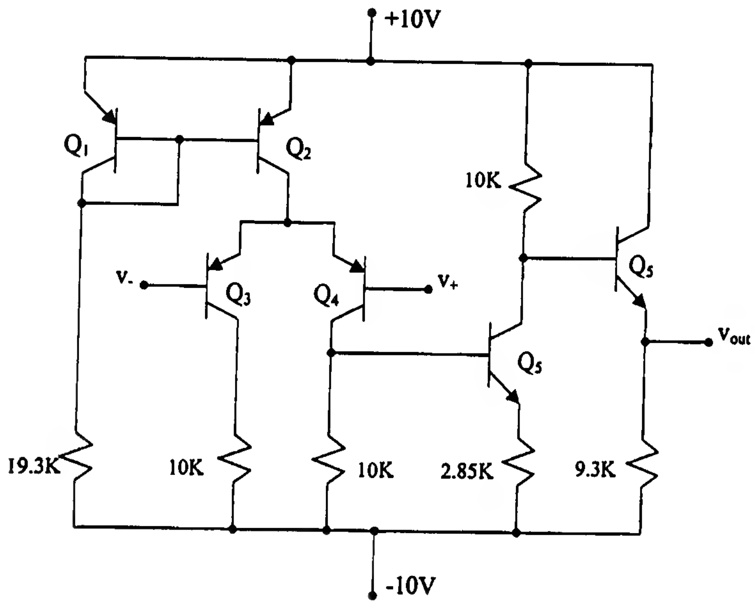
EE221
Final Examination
Open book + one formula sheet (8.5"x11")
Do 6 out of 7 questions

Instructor: Anh V. Dinh
Date: December 21, 2000
Time: 2:00pm-5:00pm
Number of pages: 4

"MERRY CHRISTMAS"

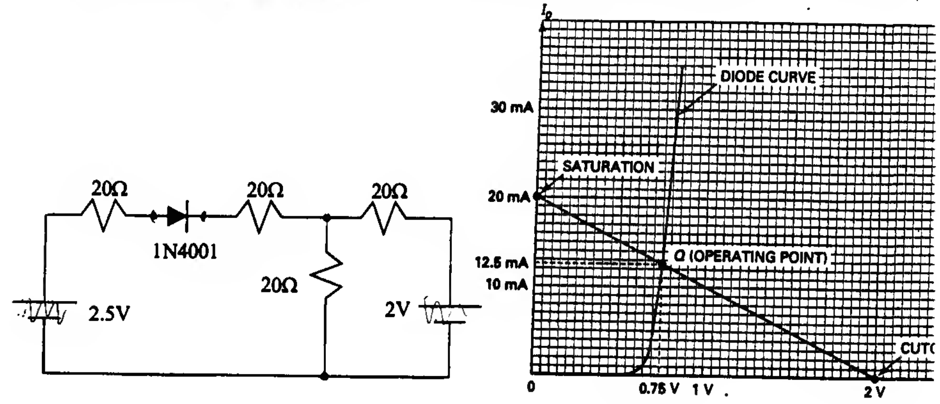
1. **Question 1**

All of the transistors in the simple operational amplifier below are matched and have an ac current gain, β, of 200 and V_{BE}=0.7V. Find the input impedance, output impedance and the open loop gain of the op-amp.



2. **Question 2**

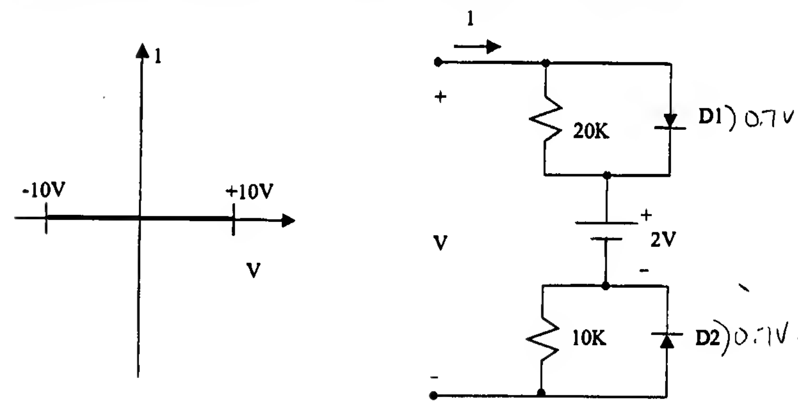
The 1N4001 diode in the figure (a) below has a VI characteristic shown in figure (b) (Figure 3-16, page 82, Malvino). Find the voltage drops across and the current through the diode. *Hint: use Thevenin circuits and the load line.*



(a) (b)

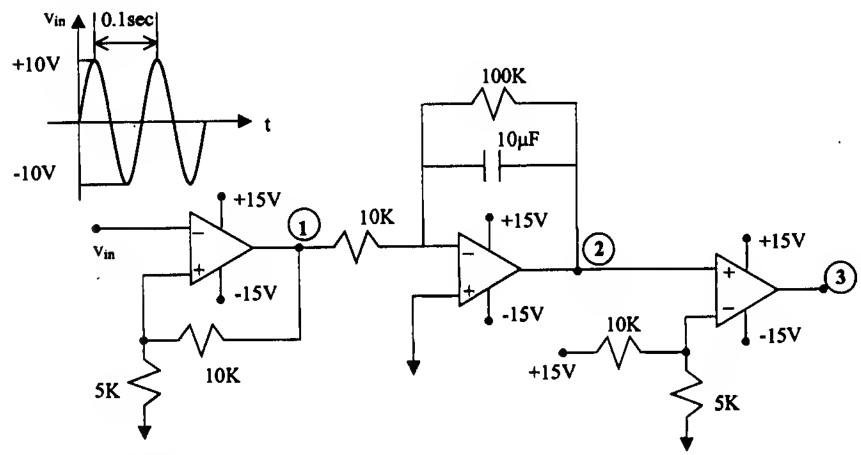
3. **Question 3**

Plot the I-V relation of the circuit shown below if each diode has a voltage drop V_D=0.7V. V_{in} varies between –10V to +10V.



4. **Question 4**

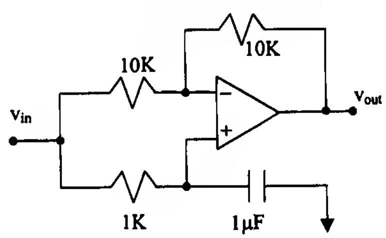
The op-amps below have saturated output voltage of $+V_{CC}=+15V$ and $-V_{EE}=-15V$. Find the peak voltages and sketch the output waveforms at nodes 1, 2 and 3 where $v_{in}=10\sin(2\pi 10t)$. What is the duty cycle of the waveform at node 3?



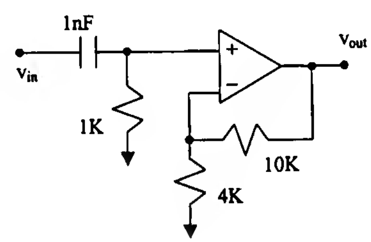
5. **Question 5**

* For the circuit Figure (a), find the delay (in degrees and time) of the output waveform with respect to the input waveform $v_{in}=0.5\sin(2\pi 160t)$. Sketch both waveforms.

* For the circuit Figure (b), write the expression of the output voltage (v_{out}) for the input voltage v_{in} of:
   a. $v_{in}=0.5\sin(2\pi 160000t)$
   b. $v_{in}=0.5\sin(2\pi 16000t)$



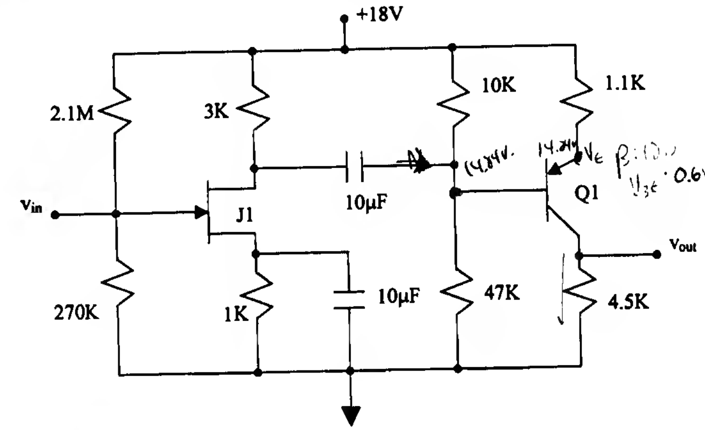
(a)



(b)

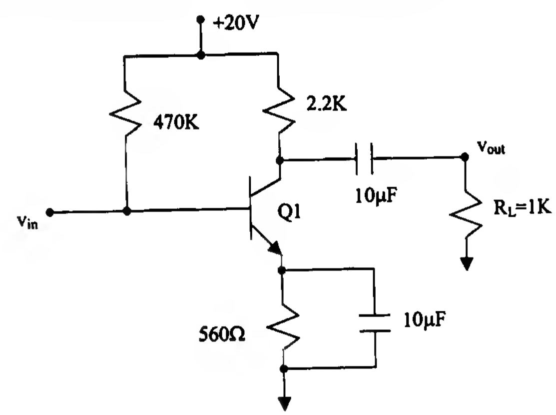
6. **Question 6**

Find input impedance, output impedance and midband gain of the multistage amplifier below. The PNP transistor, Q1, has $\beta=120$, $V_{BE}=0.6V$ and the JFET, J1, has $I_{DSS}=8mA$, $V_P=-4V$. Find low cutoff frequencies of the amplifier and sketch its frequency response. The high cutoff frequency $f_2=1MHz$.



7. **Question 7**

Find input impedance, output impedance and midband gain of the CE amplifier below. The transistor has $\beta_{DC}=\beta_{ac}=120$ and $V_{BE}=0.7V$. Sketch frequency response of the amplifier. The high cutoff frequency f2=1MHz. (*Hint: apply KVL, KCL to find currents I_B and I_E).*



University of Saskatchewan
Department of Electrical Engineering
EE 221 – Electronics I
Midterm Examination

February 2000

Instructor: J.G. Huff

Time: 2 Hours

Note: Open book
All questions of equal value
List all assumptions

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1. The circuit shown in Figure 1(a) is being supplied by an external source. The IV characteristics for the diodes used in Figure 1(a) are shown in Figure 1(b). The voltage across R1 is known to be 0.2 volts with the polarity shown. Determine values for VA and IA. Scale values from graph if necessary.

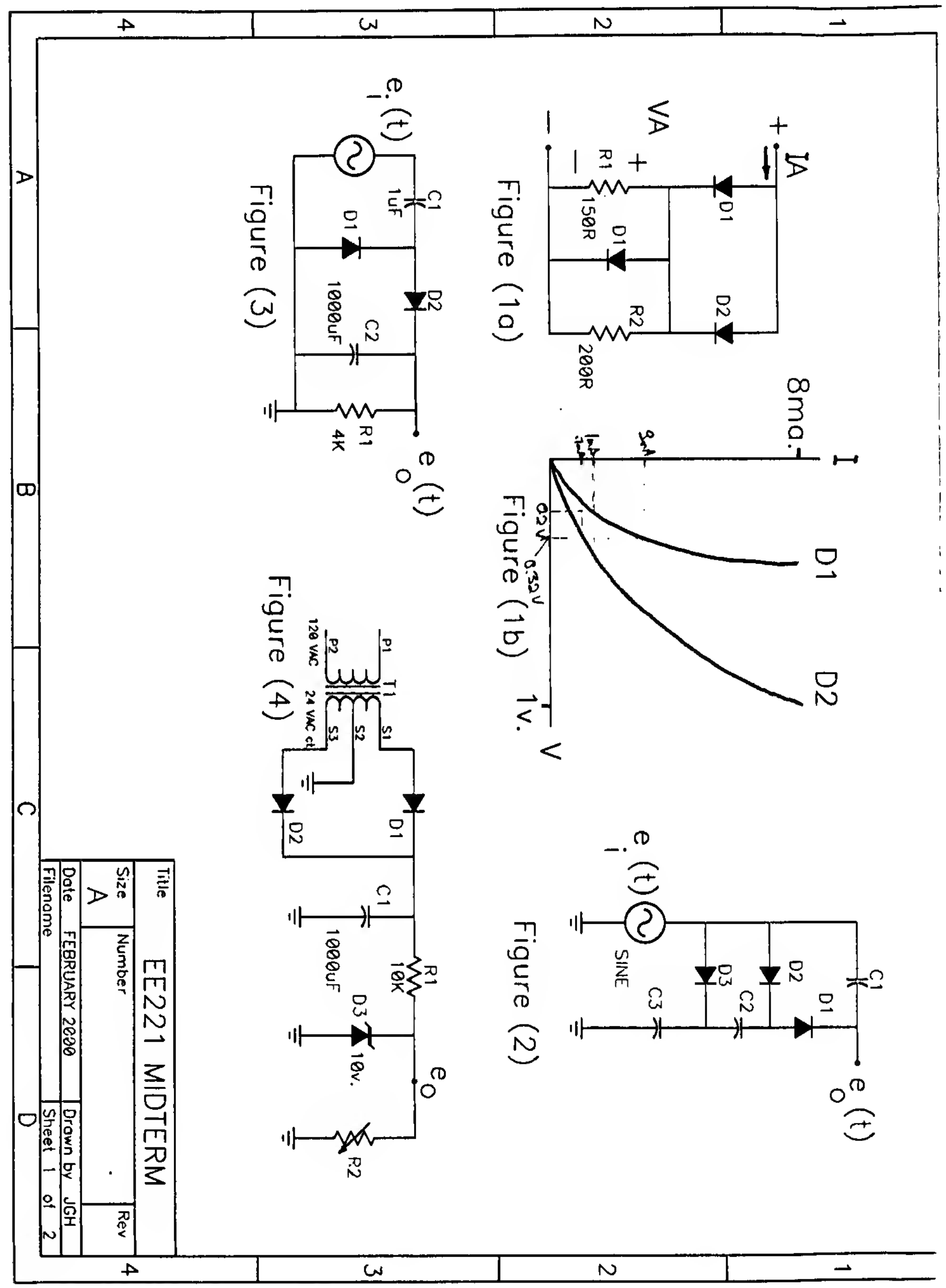
2. For the circuit shown in Figure (2), sketch $e_o(t)$ given that $e_i(t)$ is a 10 V peak to peak sine at 1 KHZ.

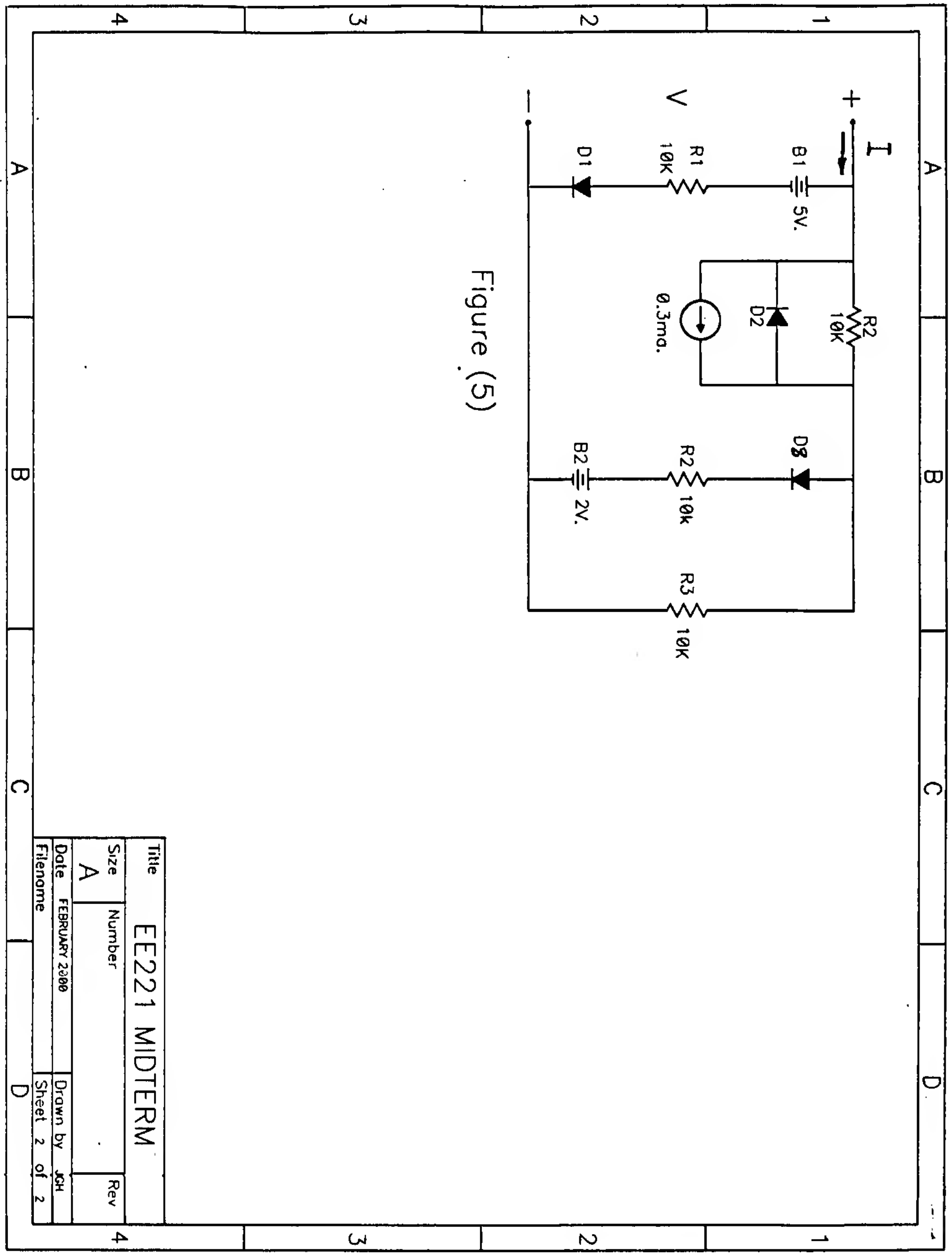
3. For the circuit shown in Figure (3), $e_o(t)$ is known to be 4 V D.C. (negligible ripple). $e_i(t)$ is a 20 V peak to peak sine wave. What is the frequency of $e_i(t)$?

4. For the power supply circuit shown in Figure (4), sketch the magnitude of e_o vs. R2 as R2 is varied from $0 \rightarrow \infty$. Assume that there is zero ripple across C1.

5. Sketch the IV characteristic for the circuit shown in Figure (5).

The End





University of Saskatchewan
Department of Electrical Engineering
EE 221 – Electronics I
Final Examination

April 2000

Instructor: J.G. Huff

Time: 3 Hours

Note: Open book
All questions of equal value
List all assumptions
All components are ideal unless stated otherwise.

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1. For the circuit shown in Figure (1), calculate the collector current I_{CQ} for transistor Q2. All transistors are 2N4124 (Data sheet attached).

2. For the circuit shown in Figure (2), calculate the actual value of V_{CEQ}. (Note: Since β is only 5, some approximations may not be valid). Assume $I_{CAO} = 0$ and $V_{AE} = 0.7$v.

3. For the circuit shown in Figure (3), both transistors are ideal with $V_{BE} = 0.7$v, $I_{cao} = 0$ and $\beta = 100$. V_{CEQ} for Q2 is known to be 8V. Determine the value of R_{a1}.

4. For the circuit shown in Figure (4), at 20^{0}C, I_{C2} is known to be 100ma. Transistors Q1 and Q2 are identical with $I_{CBO} = 1\mu a$ and $V_{AE} = 0.5$V at 20^{0}C. Calculate the value of R_a. If the temperature of the circuit was increased by 40 Centigrade degrees, calculate I_{C2} at this new temperature.

5. For the two stage amplifier shown in Figure (5), calculate the input impedance R_i, the voltage gain A_v and the output impedance R_o. Assume that both transistors are properly biased.

6. Sketch the frequency response of the amplifier shown in Figure (6). Assume that the transistor is properly biased with $I_{CQ} = 1$ma.

7. For the circuit shown in Figure (7), sketch $e_{01}(t)$ and $e_{02}(t)$ on the same time scale.

*** The End ***

2N4123
2N4124

CASE 29-02, STYLE 1
TO-92 (TO-226AA)

GENERAL PURPOSE TRANSISTOR

NPN SILICON

MAXIMUM RATINGS

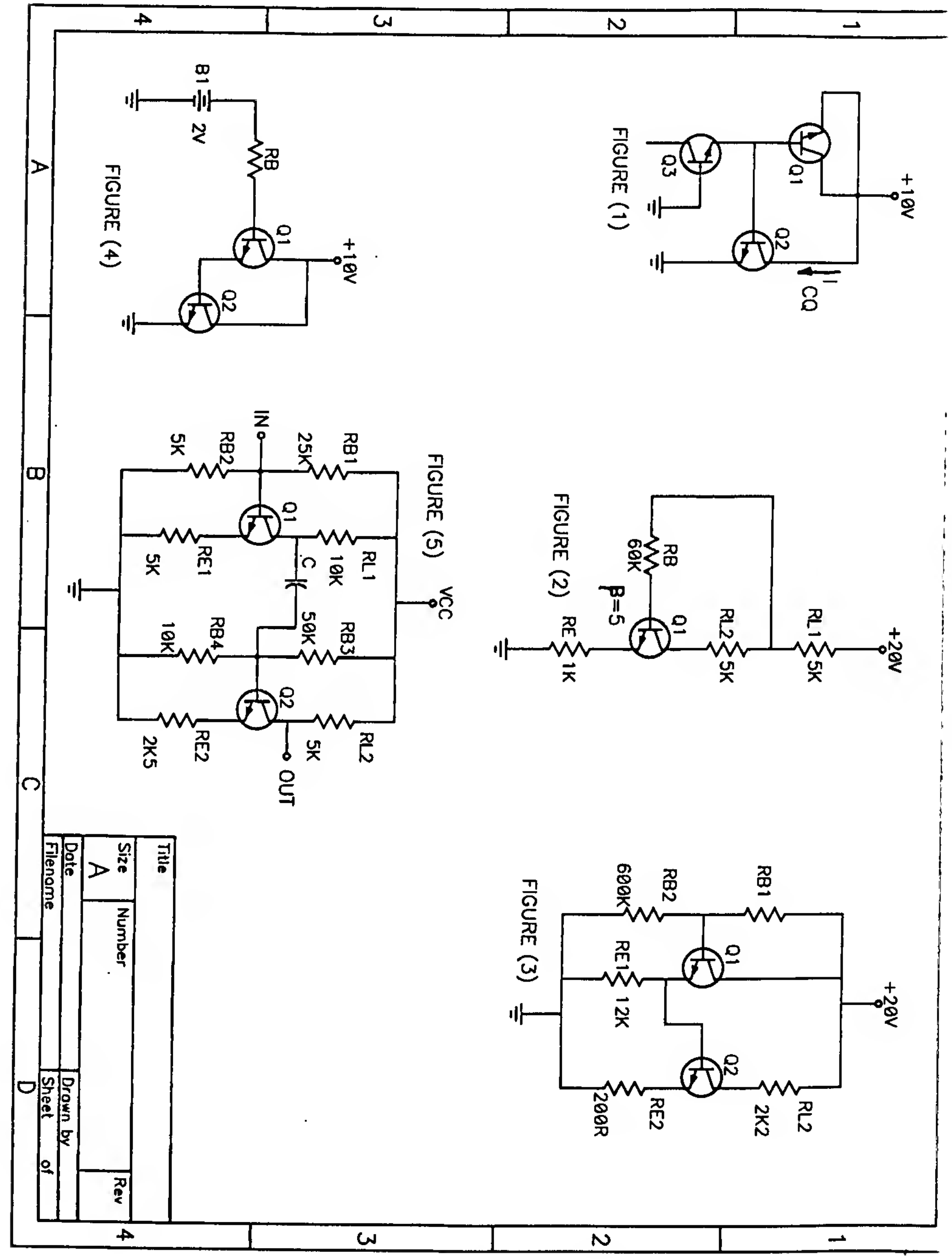
Rating	Symbol	2N4123	2N4124	Unit
Collector-Emitter Voltage	V_{CEO}	30	25	Vdc
Collector-Base Voltage	V_{CBO}	40	30	Vdc
Emitter-Base Voltage	V_{EBO}	5.0		Vdc
Collector Current — Continuous	I_C	200		mAdc
Total Device Dissipation @ $T_A = 25°C$ Derate above 25°C	P_D	625 5.0		mW mW/°C
Total Device Dissipation @ $T_C = 25°C$ Derate above 25°C	P_D	1.5 12		Watt mW/°C
Operating and Storage Junction Temperature Range	T_J, T_{stg}	−55 to +150		°C

THERMAL CHARACTERISTICS

Characteristic	Symbol	Max	Unit
Thermal Resistance, Junction to Case	$R_{\theta JC}$	83.3	°C/W
Thermal Resistance, Junction to Ambient	$R_{\theta JA}$	200	°C/W

ELECTRICAL CHARACTERISTICS ($T_A = 25°C$ unless otherwise noted.)

Characteristic		Symbol	Min	Max	Unit
OFF CHARACTERISTICS					
Collector-Emitter Breakdown Voltage(1) ($I_C = 1.0$ mAdc, $I_B = 0$)	2N4122 2N4124	$V_{(BR)CEO}$	30 25	— —	Vdc
Collector-Base Breakdown Voltage ($I_C = 10$ μAdc, $I_E = 0$)	2N4122 2N4124	$V_{(BR)CBO}$	40 30	— —	Vdc
Emitter-Base Breakdown Voltage ($I_E = 10$ μAdc, $I_C = 0$)		$V_{(BR)EBO}$	5.0	—	Vdc
Collector Cutoff Current ($V_{CB} = 20$ Vdc, $I_E = 0$)		I_{CBO}	—	50	nAdc
Emitter Cutoff Current ($V_{BE} = 3.0$ Vdc, $I_C = 0$)		I_{EBO}	—	50	nAdc
ON CHARACTERISTICS					
DC Current Gain(1) ($I_C = 2.0$ mAdc, $V_{CE} = 1.0$ Vdc)	2N4123 2N4124	h_{FE}	50 120	150 360	—
($I_C = 50$ mAdc, $V_{CE} = 1.0$ Vdc)	2N4123 2N4124		25 60	— —	
Collector-Emitter Saturation Voltage(1) ($I_C = 50$ mAdc, $I_B = 5.0$ mAdc)		$V_{CE(sat)}$	—	0.3	Vdc
Base-Emitter Saturation Voltage(1) ($I_C = 50$ mAdc, $I_B = 5.0$ mAdc)		$V_{BE(sat)}$	—	0.95	Vdc
SMALL-SIGNAL CHARACTERISTICS					
Current-Gain — Bandwidth Product ($I_C = 10$ mAdc, $V_{CE} = 20$ Vdc, $f = 100$ MHz)	2N4122 2N4124	f_T	250 300	— —	MHz
Output Capacitance ($V_{CB} = 5.0$ Vdc, $I_E = 0$, $f = 100$ MHz)		C_{obo}	—	4.0	pF
Input Capacitance ($V_{BE} = 0.5$ Vdc, $I_C = 0$, $f = 100$ kHz)		C_{ibo}	—	8.0	pF
Collector-Base Capacitance ($I_E = 0$, $V_{CB} = 5.0$ V, $f = 100$ kHz)		C_{cb}	—	4.0	pF
Small-Signal Current Gain ($I_C = 2.0$ mAdc, $V_{CE} = 10$ Vdc, $f = 1.0$ kHz)	2N4122 2N4124	h_{fe}	50 120	200 480	—



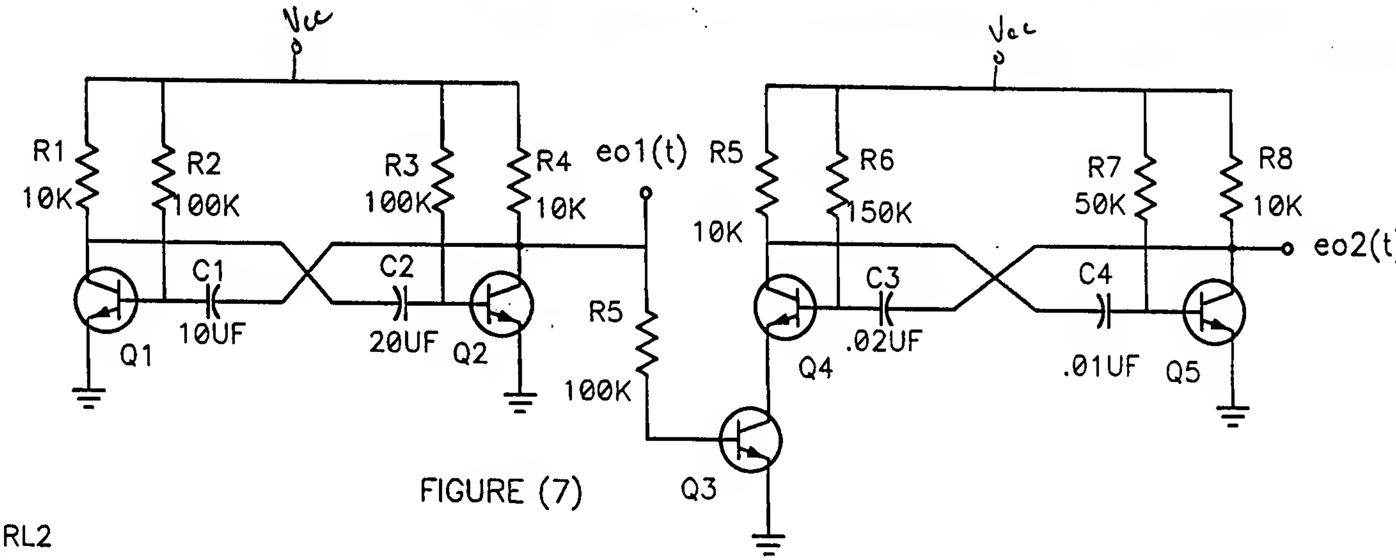


FIGURE (7)

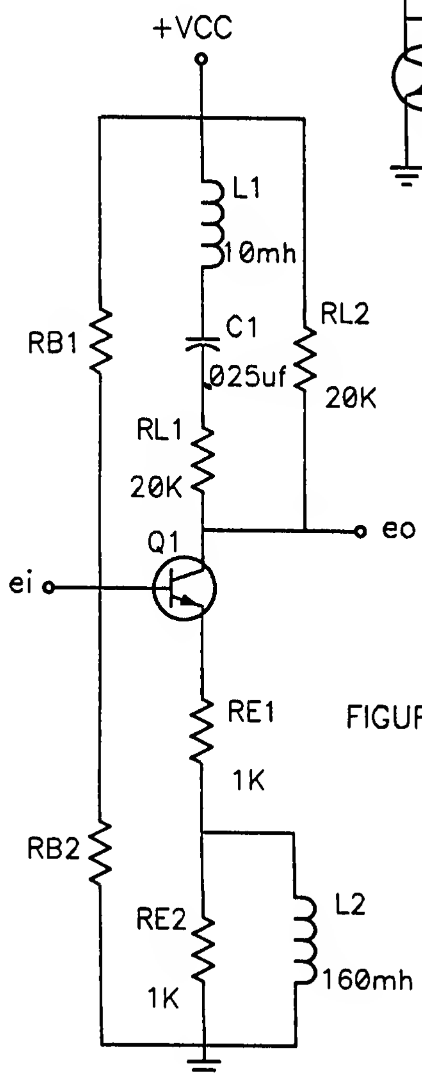


FIGURE (6)

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Size A	Number	Rev
Date		Drawn by
Filename		Sheet of

University of Saskatchewan
Department of Electrical Engineering

EE 221 Electronics I
Final Examination

April 1998

Instructor: J.G. Huff
Time: 3 hours
Note: Open Book
All questions are of equal value
List all assumptions
All components are ideal unless stated otherwise

---

1. For the circuit shown in Figure (1), ei(t) is a 20V. p-p sine wave with an average value of 5 V.D.C. Sketch $e_0(t)$.

2. For the circuit shown in Figure (2), sketch the IV characteristic.

3. The Power Supply shown in Figure (3) was designed to operate from a line voltage of 120 V.A.C. What is the minimum value that the line voltage can be reduced to without causing a decrease in the regulated 12V output of the supply?

4. For the circuit shown in Figure (4), all diodes are identical with $I_S = 10\mu a$ at 25°C. Q1 has $I_{CBO} = 10\mu a$, $\beta = 100$ and $V_{BE} = .5V$ at 25°C. Estimate a value for e_o at 35°C.

5. For the circuit shown in Figure (5), calculate a value for R_B that places Q1 in the middle of its linear operating region.

6. For the two stage amplifier shown in Figure (6), calculate the input impedance Ri, the voltage gain Av and the output impedance Ro. Assume that the transistors are properly biased.

7. Sketch the frequency response of the amplifier shown in Figure (7).

8. On a common time axis, sketch the voltage waveforms e_{1b} and e_{2b} at the bases of the two transistors. What is the frequency of oscillation of this ASMV?

*** The End ***

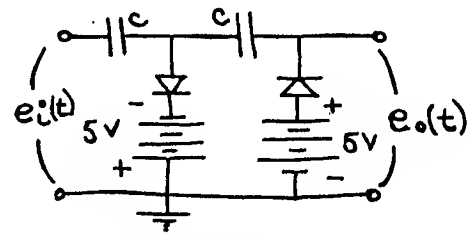


FIGURE (1)

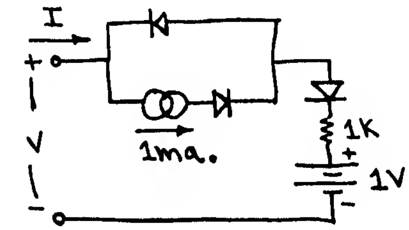


FIGURE (2)

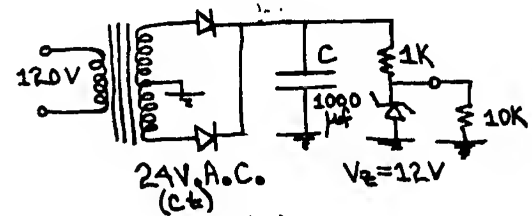


FIGURE (3)

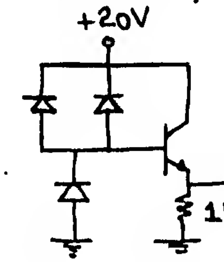


FIGURE (4)

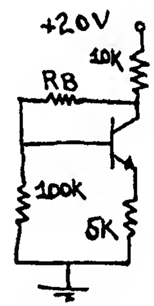


FIGURE (5)

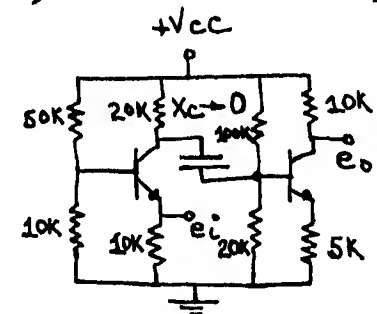


FIGURE (6)

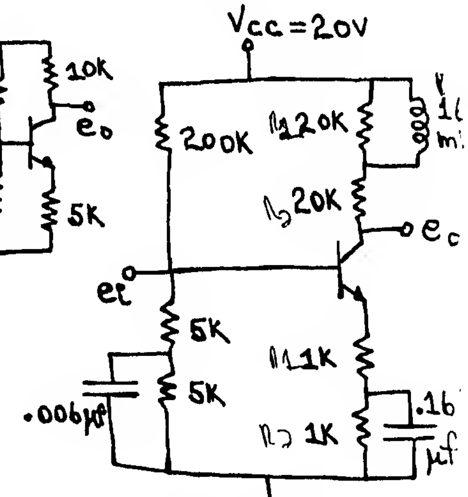


FIGURE (7)

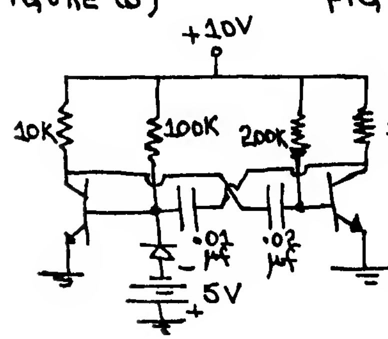
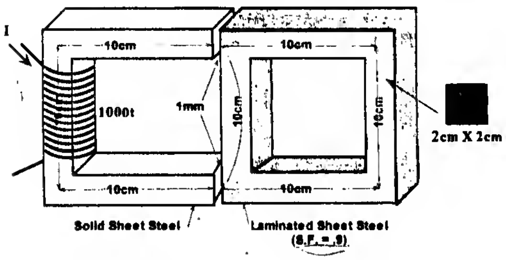


FIGURE (8)

1) A magnetic circuit is shown in the accompanying figure. The 'C' section has a coil of 1000 turns and is made of solid sheet steel. It is positioned against a square section made of laminated sheet steel (stacking factor = 0.9), and is separated from the C section by 1mm air gaps as shown. The average path length of each straight section is 10cm, and all cross-sections are 2cm X 2cm. (Note: ignore fringing effects.)



[3] a) Draw the 'electric equivalent' circuit. Label the elements including the source, NI, and the air gap(s).

3

[5] b) What current is needed in the 1000 turn coil to establish a flux of 0.56 mWb in each of the air gaps?

(Some B-H data for Sheet Steel can be found on the charts on the last page.)

(Note: Use the following table to assist with your calculations)

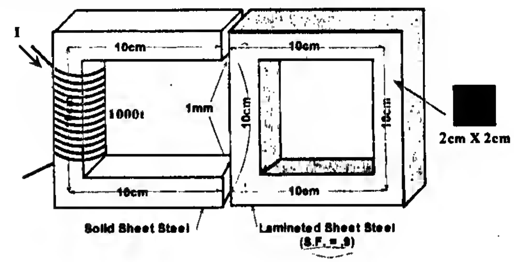
Leg	Φ (Wb)	A (m²)	B (T)	L (m)	H (At/m)	HI (At)
[illegible]	.56e-3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]		[illegible]			[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]
[illegible]		[illegible]			[illegible]	[illegible]
[illegible]		3.6x10⁻⁴		[illegible]		

2

[3] c) Once this flux is established, how much force would be required to separate the 'C' section from the laminated square?

EE 201.3 (03) Midterm Examination October 22, 2002

1) A magnetic circuit is shown in the accompanying figure. The 'C' section has a coil of 1000 turns and is made of solid sheet steel. It is positioned against a square section made of laminated sheet steel (stacking factor = 0.9), and is separated from the C section by 1mm air gaps as shown. The average path length of each straight section is 10cm, and all cross-sections are 2cm X 2cm. (Note: ignore fringing effects.)



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3

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(Note: Use the following table to assist with your calculations)

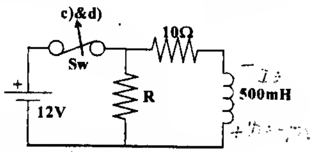
Leg	Φ (Wb)	A (m²)	B (T)	L (m)	H (At/m)	HI (At)
[illegible]	.56e-3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]		[illegible]		[illegible]	[illegible]
[illegible]	0.56e-3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	22.5
[illegible]	[illegible]	3.6x10⁻⁴	[illegible]	[illegible]	[illegible]	[illegible]

2

[3] c) Once this flux is established, how much force would be required to separate the 'C' section from the laminated square?

Student #: Page 2 of 7

7) Consider the R – L circuit in the accompanying figure. The switch has been closed long enough so that steady state conditions have been reached.



[2] a) What current is flowing through the inductor?

[2] b) How much energy is stored in the inductor?

[4] c) What is the maximum value of the resistor, R, that will ensure that the voltage across the switch contacts will not exceed 200V when the switch is opened?

[4] d) The manufacturer notes that the switch will not be damaged by arcing as long as the voltage across its open terminals does not exceed 500V for more than 500μsec. If a resistor, R = 1KΩ is used, will this requirement be met?

[2] 8) If $v(t) = 170\sin(377t)$ V,

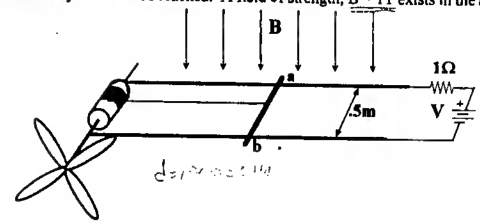
a) what is the frequency, f? ________

b) what is the period, T? ________

[1] 4) Write the expression for a sinusoidal waveform, $v(t)$, for which the peak to peak voltage, V_{P-P} = 24 Volts and the period, T = 20.0 msec.

____

5) A bar and rail are arranged to spin a small fan as shown in the figure. The fan requires 15W of power to turn it at 600rpm. The cable is attached to a drum that is 10cm in diameter. Assume that the spool of cable and the length of the rails are both long enough so that steady state can be reached. A field of strength, B = 1T exists in the area.



[1] a) At what velocity must the bar move to spin the fan at the required speed?

[1] b) What force must be applied to the cable (by the bar) to deliver the required power?

[2] c) How much current is required to supply the necessary force, and what is its direction through the bar (a⇒b or b⇒a)?

[2] d) What is the motional electromotive force generated by the bar and what is its polarity (a or b positive)?

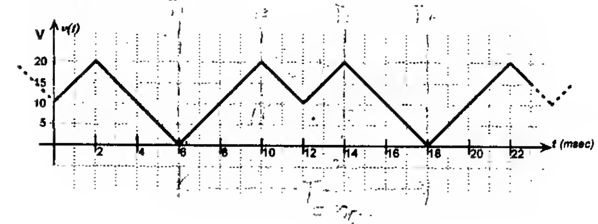
[2] e) If the bar also had an internal resistance of 1Ω, what supply voltage, V, would be required to provide the necessary current?

[2] f) What is the overall efficiency of the system?

[1] 6) What is the frequency of a sinusoidal waveform that completes 42 cycles in 360?

f= ________

9) Determine the a) average and b) effective (RMS) values for the waveform shown below.



[4]

Area ... = 0.07 Vs

Average = Area / period = 0.07 Vs / ... = $\boxed{11.67\text{ V}}$

[6]

$I^2 = \int ...$

$I^2 = ...$

$I^2 = \frac{25}{...}$ ... $+ \frac{25}{3} t^3 \Big|_{-4}^{0}$

$I = \sqrt{... + \frac{25(-8)}{3} ...}$

$\boxed{I_{rms} = 44.7\text{ V}} > V_{max}$??

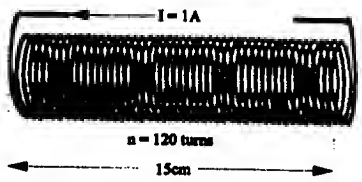
10) If $I(t) = 15\sin(377t + \pi/6)$ A,

[1]

a) what is the value of the current at t = 16.67 msec? 7.5 A

Question 1:

The tightly wound, air-core coil shown in the figure is excited by a current of 1A which produces a flux of 0.113μWb inside the coil.



[2] i) Calculate the inductance of the coil.

$L = \frac{N\Phi}{I} = \frac{(120\text{ turns})(0.113\mu Wb)}{1A} = 13.56\mu H$ ✓

2

$\frac{l}{d} \geq 10 \quad = \frac{.15m}{.01196}$

[2] ii) Estimate the diameter of the coil.

$L = \frac{\mu_0 N^2 A}{l} \quad A = \frac{Ll}{\mu_0 N^2} = \frac{(13.56\mu H)(.15m)}{(4\pi\times10^{-7})(120)^2}$

$= 1.124\times10^{-4}\,m^2$

2

$A = \pi r^2 \quad r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{1.124\times10^{-4}}{\pi}}$

$= 5.982m$ ✓

$\therefore d = 2r = 0.01196m$

[1] iii) Will the inductance increase or decrease if an iron core is placed inside the coil?

The inductance will increase.

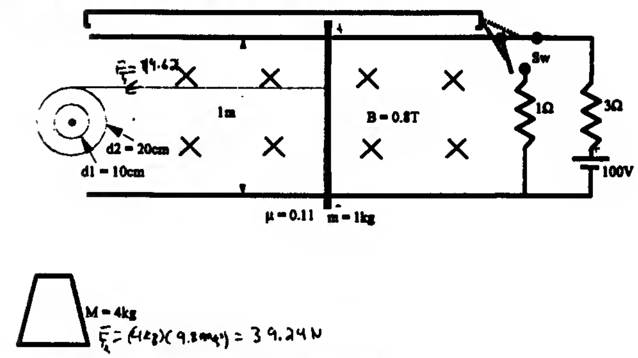
|

Student Number:

5

Question 2:

A bar (with no internal resistance) rests on two rails 1meter apart. A cable attached to the bar is wound around a cylinder with a diameter of 20cm. A weight with mass = 4kg is attached to another cable which is wound around another cylinder with a diameter of 10cm which is fixed to the same axel. A 100V battery and resistors are available to be connected to the rails through a switch arrangement. Assume the weight and friction from the cables as well as wind resistance etc. are negligible. Assume the bar is at rest before the switch is closed to connect the 100V battery to the rails through the 3Ω resistor. (Refer to the figure below.)



$F_t = (4kg)(9.8m/s^2) = 39.24N$

a) Immediately after the switch is closed (i.e. $t=0^+$), determine:

[1] i) the magnitude and direction of the current through the bar

$V = IR \quad I = \frac{V}{R} = \frac{100V}{3\Omega} = 33.33A$ counter clockwise ✓ (ccw)

/

[1] ii) the motional EMF of the bar, E_m

because the bar is not moving

$E_m = 0V$ ✓

|

[1] iii) the magnetic force on the bar, F_{mag}

$F = Il \times B$

$= (33.37A)(1m)(.8T)$

$= 26.66N$ ✓ to the right

|

Student Number:

3

[1] iv) indicate if the bar will move and in which direction.

The bar will move to the right.

$F_{r_1} < F_{mg}$

$\tau = (84.24N)(.10m) = 3.934N$

$(8.424N) = F(.20...)$

$F_r = 19.62$

b) Assume the rails are long enough for the system to reach "steady state" and determine:

[1] i) the magnitude and direction of the current through the bar

$F_{r_1} + F_{Em} = F_{mg}$

$(19.62N) F_{Em} = 26.66N$

$F_{Em} = 7.04N$

$F = I l \times B$

$I = \frac{F}{l \times B} = \frac{7.04N}{(1m)(0.8T)}$

$= 8.8A$

[1] ii) the motional EMF of the bar, E_m

$E_m = 100V - (8.8A)(3\Omega)$

$= 73.6V$

1/2

[1] iii) the magnitude and direction of the velocity of the bar

$E_m = l v_d \times B$

$v_d = \frac{E_m}{l B} = \frac{73.6V}{(1m)(.8T)} = 92 m/s$ to the right.

1/2

[1] iv) whether the bar is acting as a motor or a generator.

motor.

c) When the bar reaches the end of the rails, it activates the switching arrangement such that the battery is *disconnected* from the system and a 1Ω resistor is now connected across the rails. Again assume the rails are long enough for the system to reach steady state and determine:

[1] i) the magnitude and direction of the current through the bar

$F_{Em} = F_{r_1} = 19.62N - F_{Fr}$

$F = I l \times B \Rightarrow I = \frac{F}{l B} = \frac{19.62N}{(1m)(.8T)}$

$I = 24.52A$ counter clock wise (CCW)

1/2

[1] ii) the magnitude and direction (up or down) of the velocity of the weight, M

$V = IR = (24.52A)(1\Omega) = 24.52V$

$E_m = l v_d \times B$

$v_d = \frac{E_m}{lB} = \frac{24.52V}{(1m)(.8T)} = 30.65 m/s$

mass would be moving half as much.

$\therefore v = 15.32 m/s$ down

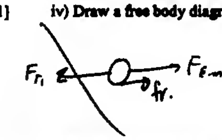
Student Number:

11/14

[1] iii) whether the bar is acting as a motor or a generator.

generator.

[1] iv) Draw a free body diagram for the bar.



d) From the data determined in part b) at steady state (battery and resistor connected):

[1] i) calculate the total *electric* power supplied to this system

$P_{Batt} = IV = (8.8A)(100V) = 880W$

1/2

[1] ii) indicate the source (i.e. where it comes from)

It is supplied from the battery.

[1] iii) identify where it is consumed (including the quantity).

It is consumed by the F_{Em} of the bar, $= (73.6V)(8.8A) = 644.4W$

and the mass $P_{mass} = F v_d = (19.62N)(92m/s) = 1805.04W$

$\therefore$ energy is wrong.

1/2

P_{Batt} 25 87.5 W

all power is dissipated ...

Student Number:

/3

e) From the data determined in part c) at steady state (only resistor connected):

[1] i) calculate the total *mechanical* power supplied to this system

$P_{mech} = (19.62\ N)(30.65\ m/s) = 60\cancel{1}.35\ W$

1/2

$m_g = 568.2\ W$

[1] ii) indicate the source (i.e. where it comes from)

· It's supplied by ~~the~~ Batteries.

[1] iii) identify where it is consumed (including the quantity).

Batteries

$P_{elec} = IV = (24.\cancel{5A})(24.52\ V) = 601.23\ W$.

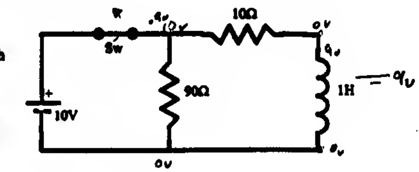
$(P_{elec} = P_{mech})$

Student Number:

1/2

Question 3:

Consider the circuit shown in the accompanying figure. Assume the switch has been closed long enough for the system to reach steady state. Determine:



[1] i) the steady state current through the inductor

$R_{th} = 9\Omega$ V_{th}

$V = IR$ $I = \frac{V}{R} = \frac{10V}{10\Omega} = 1A$

$i_L = 1A$

[1] ii) the steady state voltage across the inductor

at steady state there is no change in current and ∴ no opposition to change. ∴ there is no voltage across the inductor.

$V = 0$

[3] iii) the peak voltage that will appear across the switch contacts immediately after the switch is opened (i.e. at t=0⁺).

$V = IR$ $I = \frac{V}{R} = \frac{9V}{100\Omega} = 0.09A$

$V_L = -I_0 R_{Th} (e^{-T/\tau})$

1/2

$V_L = -I_0 R_{th} = -(1A)(9\Omega) = -9V$

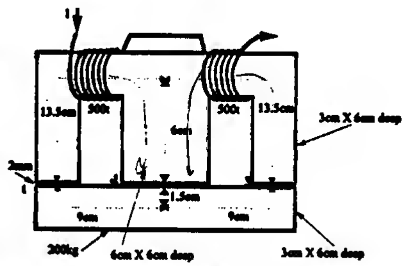
$V_{across\ switch} = 9\cancel{}V$

Student Number:

2 1/2

Question 4:

An electromagnet is use to remove hot ingots which have a mass of 200kg from a steel mill's conveyor (Note: temperature well below Curie temperature!). When attaching the electromagnet it can be brought to within 2mm of the ingots (i.e. assume a uniform air gap of 2mm). The magnet material and the ingot are both solid sheet steel. Some selected B-H data for sheet steel is shown below in Table 1. Neglect fringing at the air gaps (i.e. the air gap area is the same as the adjacent core area). Refer to the figure for other relevant data.



Some B-H data:

B (T)	0 - .2	.2 - .4	.4 - .5	.5 - .6	.6 - .7	.7 - .8	.8 - .9	.9 - 1.0	1.0 - 1.1
H (At/m)	90	100	110	120	130	140	150	175	250

Use the following table to help with your calculations:

Leg	Φ	A	B	H	l	Hl
Left leg						
centre leg						
Right leg						
gap 1	.7457 mw	.0018m	.414			
gap 2	3.139 mW	.0036m	0.872			
gap 3	.7452 mw	.0018m	.414			

[2] a) Determine the flux density required to start to lift the ingots.

$F = mg = 200 \times 9.81 = 1962 N \quad \therefore \frac{F}{2} = 981$

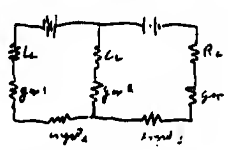
$B_g = \sqrt{\frac{2\mu_0 F}{A}} = \sqrt{\frac{2 \cdot 4\pi \times 10^{-7} \cdot 981}{(.06)(.06)}}$ ~~.828~~ OK

$B_g = 0.627T \quad \therefore \Phi = 3.139 mw$

(1½)

[2] b) Draw an "electric equivalent" for this magnetic circuit.

(2)



3½

[2] c) What is the total flux flowing in the centre leg?

from part A

$B = \cancel{.872}\ T$

(2) d) What total NI (magnetomotive force) is required to produce this amount of flux?

$H = \frac{B}{\mu} = \frac{.872T}{4\pi \times 10^{-7}} = 693915.5\ At/m$

$Hl = (H)(.06m) = 41634.9\ AT$

$Hl = NI = 416\cancel{34}.9\ AT$

½

(2) e) What current must flow through the windings to produce this flux?

$I = \frac{Hl}{N} = \frac{41634.9\ AT}{1000T} = 4\cancel{1}.63\ A$

[2] f) Once the ingot is attached (gap = 0mm), will the total flux in the centre leg increase of decrease?

It will increase

(2)

[1] g) Once the ingot is attached (gap = 0mm), will the steady state current in the coil increase of decrease? I will remain the same (as is I thought) but it could be decreased and still maintain enough force.

(1)

[2] f) If the current is supplied from a 100V ideal battery (i.e. no internal resistance), and the coils have a total resistance of 1Ω, how long will it take after the coil is energized to reach the level of current required to start to lift the ingots?

3½

$\frac{46}{50} = 92\%$

EE 201 - Electric and Magnetic Circuits II
Midterm Examination
October 26, 2000
Allowed Time: 2 Hours

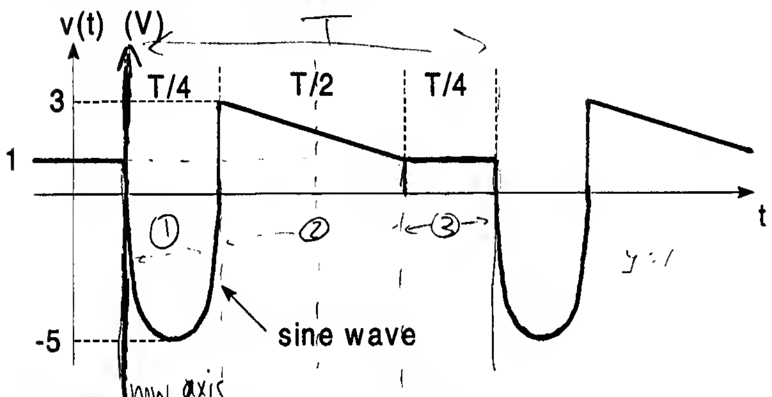
NAME: ______

______

STUDENT #: ______

Closed book examination. One 8.5" × 11" sheet of notes allowed.
Do all questions. Marks for each question are in [ ].

1. Consider the periodic voltage waveform v(t) below.



(a) What is the average value of v(t)? Show all calculations. [3] $-\frac{T}{2}$

$AVG = \frac{\text{Area under Curve}}{T} = \frac{A_1 + A_2 + A_3}{T}$

$A_2 = mx + b = \left(\frac{-2}{T/2}\right)t + 3 = \int_0^{T/2} \frac{-4}{T}t + 3 \, dt = \frac{-4t^2}{2T}\Big|_0^{T/2} + 3t\Big|_0^{T/2} = \frac{-2\left(\frac{T^2}{4}\right)}{T} + \frac{3T}{2} \quad T$

$A_3 = \left(\frac{T}{4}\right)1 = \frac{T}{4}$

$A = -5$

$A_1 = A \sin \omega t \; ; \; \omega = 2\pi f, \; f = \frac{1}{T_{sw}} \quad T_{sw} = T/2 \quad \sin\left(\frac{2\pi t}{T_{sw}}\right)$ sub in

$\int_0^{T/4} \left(\sin \frac{2\pi t}{T_{sw}}\right) dt = -\cos \frac{2\pi t}{T_{sw}}\Big|_0^{T/4} \left(\frac{T_{sw}}{2\pi}\right) = \left(\frac{T}{4\pi}\right)\left[-\cos \frac{4\pi t}{T}\right]_0^{T/4}$

$= \frac{T}{4\pi}\left[-\cos \frac{4\pi T/4}{T} + \cos 0\right] = \frac{T}{4\pi}\left[0 + 1\right] = \frac{T}{4\pi}$

$AVG: \frac{\frac{T}{4\pi} + \frac{T}{4} + T}{T} = \frac{1}{4} + 1 + \frac{1}{4\pi}$ $\boxed{AVG = \cancel{2.04}}$

(b) What is the rms (effective) value of v(t)? Show all calculations. [4]

$Rms = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$ $\quad Rms = \sqrt{\frac{1}{T}\left(\int_0 v \right.}$

2 $= \left(\frac{-4t}{T} + 3\right)^2 = \frac{16t^2}{T^2} - \frac{24t}{T} + 9$ / 3 $= \left(\frac{T}{4}\right)^2 = \frac{T^2}{16}$

2 integrate $\int_0^{T/2} \frac{16t^2}{T^2} - \frac{24t}{T} + 9 \, dt = \frac{16t^3}{3T^2}\Big|_0^{T/2} - \frac{24t^2}{2T}\Big|_0^{T/2} + 9t\Big|^{T/2} = \frac{16\left(\frac{T^3}{8}\right)}{3T^2} - \frac{24\left(\frac{T^2}{4}\right)}{2T} +$

$= \frac{2}{3}T - 3T + \frac{9T}{2} = \frac{4}{6}T - 3T + \frac{27}{6} = \frac{31T}{6} - \frac{18T}{6} = \frac{13}{6}T$

3 Area under Squared curve still $\frac{T}{4}$

$A^2 = 25$

1 $\left[A \sin\left(\frac{2\pi t}{T}\right)\right]^2 = A^2 \sin^2\left(\frac{2\pi t}{T}\right) = \frac{1}{2}\left(1 - \cos 2\left(\frac{2\pi t}{T}\right)\right) = \frac{1}{2}\left(1 - \cos\left(\frac{4\pi t}{T}\right)\right.$

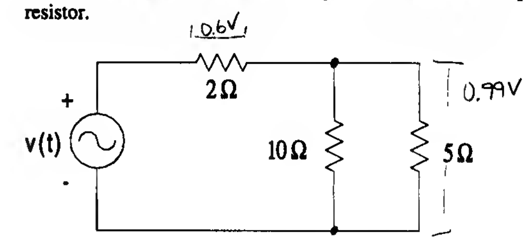
$\frac{1}{2}\int_0^{T/4} 1 - \cos \frac{4\pi t}{T} \, dt = \frac{1}{2}\left(t\Big|^{T/4} - \sin \cancel{\frac{4\pi t}{T}} \left(\frac{T}{4\pi t}\right)\right)\Big|_0^{T/4}$

$= \frac{1}{2}\left[\frac{T}{4} - \left(\frac{T}{4\pi t}\right)\sin \frac{4\pi t}{T}\Big|_0^{T/4}\right] = \frac{1}{2}\left[\frac{T}{4} - \frac{4T}{4\pi T}\sin \frac{4\pi T}{4T}\right] = \frac{T}{8} - 0 = \frac{T}{8}$

$Rms = \sqrt{\frac{13}{6}\cancel{T} + \frac{1}{4}\cancel{T} + \frac{1}{8}\cancel{T}} = \boxed{1.59\text{ V}}$ X

3

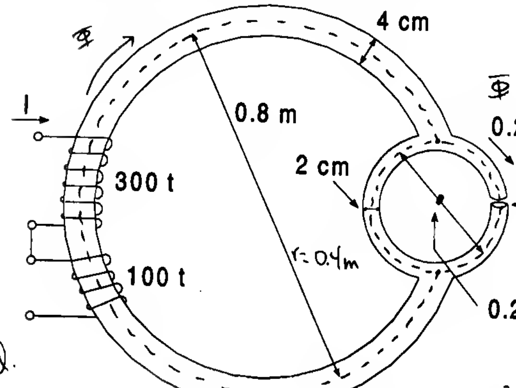
(c) If v(t) is connected to the following circuit, calculate the power dissipated by each resistor. [3]

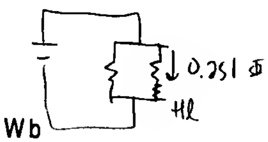


$V_{2\Omega} = \frac{V R_{2\Omega}}{R_{2\Omega} + R_{3.3\Omega}} = \frac{1.59V\,(2\Omega)}{5.3\Omega} = 0.6V$

B-H CURVES ARE ON LAST PAGE

2. The magnetic core below has been formed by joining two toroids. Each has a circular cross-section. The larger toroid joins the smaller toroid at two points such that the two paths in the smaller toroid are the same length—except, of course, for the 1 mm gap in the portion on the right. Both toroids are cast steel.





cast steel.

2cm : 1cm radius

$A = \pi r^2$

$\pi (0.7)^2$ $A = \pi (0.011)^2$

$C = \pi d$

$C = \pi 0.x$

(a) If the flux in the right leg of the smaller toroid is $\Phi = 0.251$ mWb, calculate I. Take fringing into account. [6]

Portion	Area (m^2)	Length (m)	Φ (Wb)	B (T)	H (At/m)	Hℓ (At)
Right leg (small toroid)	3.142×10^{-4}	0.3917	0.000251	0.7989	≈500	195.85
Gap	0.000121π	0.001	0.000251	0.6603	525450	525.45
Left leg (small toroid)	3.142×10^{-4}	0.3927	0.000432	1.375	1836.8	721.3
Large toroid	1.257×10^{-3}	2.2633	0.000683	0.5434	≈380	860.054

+

5

$B = \mu H$

$H = \frac{B}{\mu}$

$\boxed{I = 3.95A}$

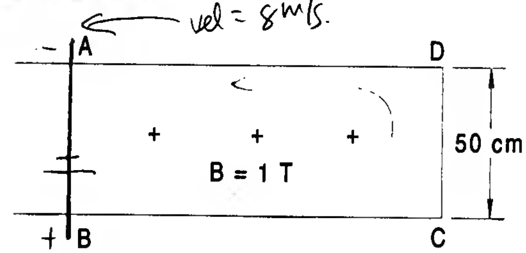
(b) Calculate the magnitude of the magnetic force trying to close the gap. [2]

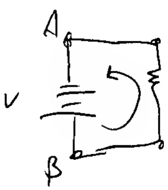
$F = \frac{B_g^2 A_g}{2\mu_0} = \frac{(0.6603)^2(0.000121\pi)}{2(4\pi\times10^{-7})}$

$\boxed{F = 65.9N}$

2

3. A conducting rod AB is in contact with metal rails AD and BC which are placed 50 cm apart in a uniform magnetic field, B = 1 T, which is directed into the page as shown. The total resistance of the circuit ABCD is 0.4 Ω and is assumed constant regardless of the rod's position. Ignore friction.





$4V = I(0.4\Omega)$

$I = 10A$

(a) What is the magnitude of the emf induced in the rod when it is moved to the left with a constant velocity of 8 m/s? Which point, A or B, is at the higher potential? [3]

$E_m = l\, vel \times B$

$E_m = (0.5m)(8m/s)(1T)$

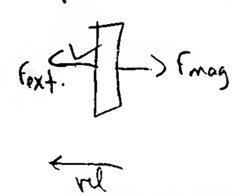
$\boxed{E_m = 4V}$

B will be at higher potential

7

(b) What external force (magnitude and direction) must be applied to the rod to keep it in motion at this velocity? [3]

FBD.



neglecting friction, $f_{ext} = F_{mag}$

$F_{mag} = l I \times B$

$F_{mag} = (0.5m)(10A)(1T)$

$F_{mag} = 5N$ right

$\boxed{\therefore f_{ext} = 5N \text{ left}}$

4

(c) Calculate the electric power dissipated in the circuit, and compare it to the mechanical power being done by the external force. [3]

electrical

$P = I^2R$

$P = (10A)^2(0.4\Omega)$

$P = 40W$

mechanical.

$P = (F_{mag})(vel)$

$P = (5N)(8m/s)$

$P = 40W$

$\boxed{P_{elect.} = P_{mech} \quad 40W = 40W}$

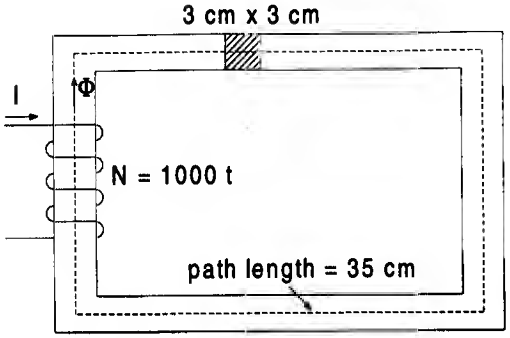
3

(d) Is the rod acting as a motor or a generator? Why? [1]

rod is acting as a generator because there is

4. Calculate the flux, Φ, flowing in the cast iron core below if I = 0.945A and N = 1000 turns. [5]

10



$l = 0.35m$
$I = 0.945A$
$N = 1000$
$\Phi = ?$

$NI = Hl$
$945 = 945$

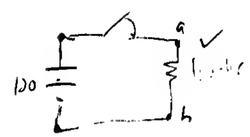
Cross-Sect. Area (m^2)	Length (m)	Φ (wb)	B(T)	H (At/m)	Hl (At)
0.0009	0.35m	?	0.61	2700	945

$\Phi = BA$

$\Phi = (0.61\,T)(0.0009)$

$\boxed{\Phi = 0.549 \text{ mWb}}$

5



5. Select the correct word to complete the following statements: [5]

(a) Switching on a toaster (does/doesn't) change the voltage at the outlet.

(b) Switching off the lights makes it (easier/harder) for SaskPower to turn their generators.

(c) To calculate average power dissipated in an ac circuit, you must use the (average/effective) voltage or current.

(d) The voltage across an inductor is proportional to the (integral/derivative) of the current through it.

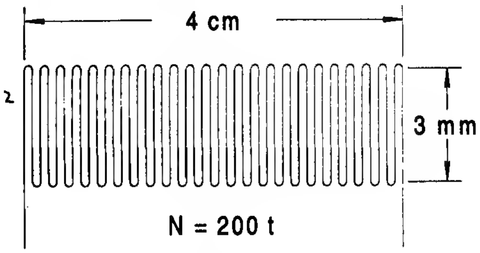
(e) If you consume more electricity (power), SaskPower must burn (less/more) coal to turn their generators.

5

$V_L = L\frac{di}{dt}$

6. What is the inductance of the air-core coil below? The coil has a circular cross-section. [5]

10



$A = \pi(0.0015)^2$
$l = 0.04m$
$\mu_0 = 4\pi \times 10^{-7}$
$N = 200t$

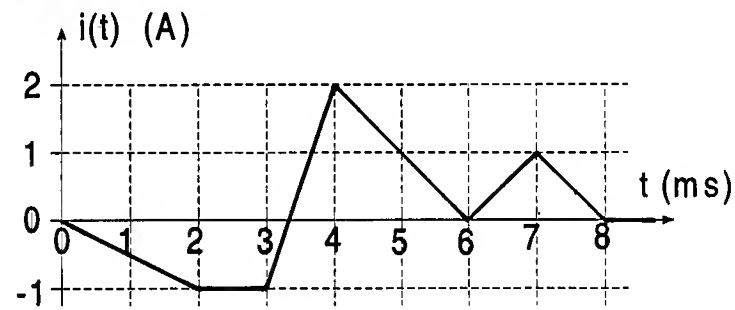
$L = \frac{\mu_0 N^2 A}{l}$

$L = \frac{(4\pi \times 10^{-7})(200)^2(\pi \cdot 0.0015^2)}{0.04m}$

$\boxed{L = 8.88\ \mu H}$

5

7. The current through a 15 mH inductor is shown below. Calculate and sketch the voltage across the inductor. [5]



$V_L = L\frac{di}{dt}$

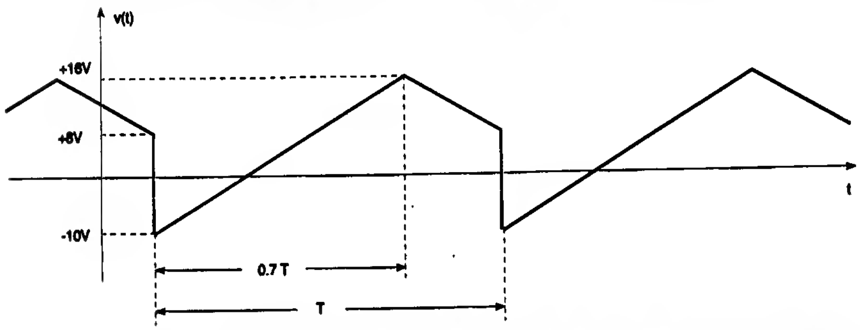
Time	di/dt	=	* 15 mH
0-2 ms	$\frac{-1}{2}$	-0.5 A/ms	-7.5
2-3 ms	0		0
3-4 ms	3 A/1 ms	3	45
4-6 ms	-2A/2s	-1	-15
6-7 ms	1A/1ms	1	15

5

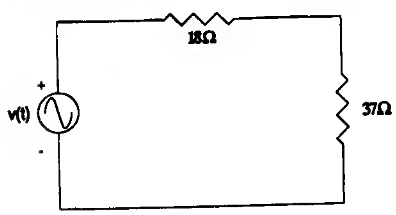
GE 212 - Electric and Magnetic Circuits II
Midterm Examination
October 26, 1999
Allowed Time: 2 Hours

Closed book examination. One 8.5" × 11" sheet of notes allowed. All questions equal value. Do all questions. Marks are in [ ].

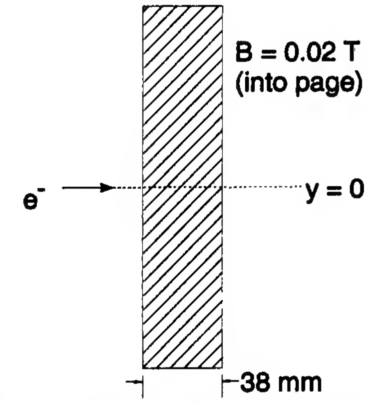
1. (a) What is the average value of the periodic voltage waveform below? [4]



(b) A D'Arsonval meter with a single series diode that has been calibrated to read the RMS value of AC sinewaves is used to measure this voltage waveform. What are the two possible readings? [6]
(c) What is the RMS value of the waveform? [6]
(d) If this voltage waveform is connected to the following circuit, determine the average power dissipated by *each* of the resistors. [4]

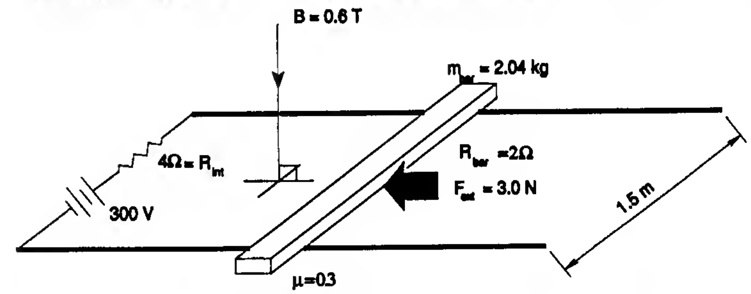


2. An electron travelling at a velocity of 2×10^8 m/s enters a region of magnetic flux 38 mm wide, as shown on the following page. The mass of an electron is 9.11×10^{-31} kg.



(a) In what direction will the electron's path bend, up or down? [3]
(b) What is the radius, r, of the electron's path while in the region of the magnetic field? [3]
(c) At what position will the electron emerge from the magnetic field? (in other words, y = ?) [9]
(d) Describe where the electron would emerge from the magnetic field if B was increased to 0.06 T. [5]

3. The bar shown below has a mass of 2.04 kg and slides with a constant velocity on rails spaced 1.5 m apart. The 300 V DC supply has an internal resistance of 4 Ω as indicated. The rails have negligible resistance while the bar has an internal resistance of 2 Ω. The dynamic coefficient of friction is 0.3. There is a uniform perpendicular magnetic field, B = 0.6 T, acting downward throughout the region between the rails. A constant external force of 3.0 N acts on the bar to the left.



(a) Draw the electrical equivalent circuit and the free body diagram of the setup. [3]
(b) Determine:

- direction of motion of the bar (explain your answer) [3]
- direction and magnitude of the current in the circuit [2]
- velocity of the bar [2]
- polarity and magnitude of the voltage between the rails [2]
- is the bar acting as a motor or a generator (explain)? [2]
- find the efficiency of the system [2]

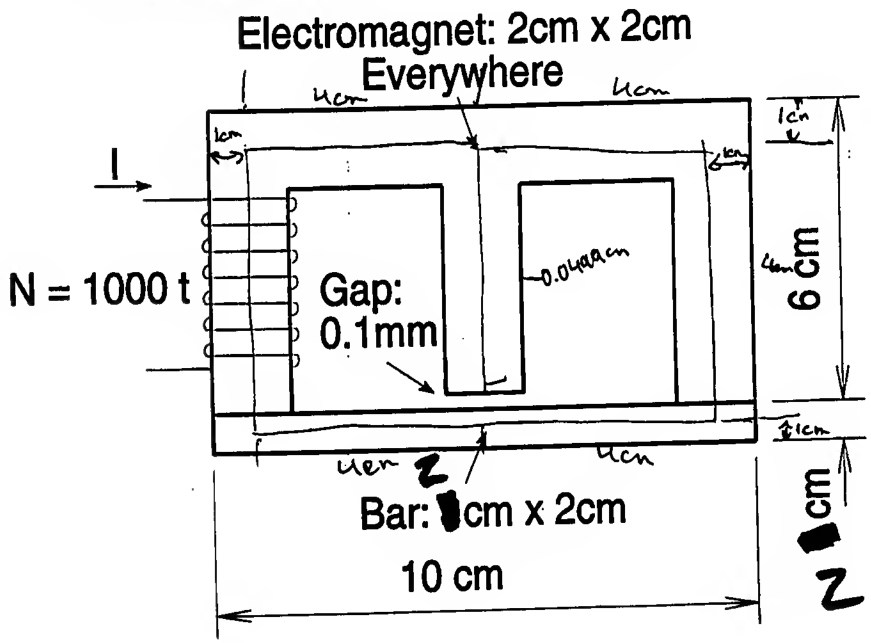
(c) Explain in detail what would happen if the direction of the magnetic field is suddenly (almost instantaneously) flipped 180°. [4]

4. A ***cast steel electromagnet*** has a 1000 turn coil on its left leg as shown below. A magnetic flux of 120 μWb is required in the gap to provide the necessary force to hold the ***cast steel bar*** in place against the electromagnet. The core is perfectly symmetrical with the center leg located precisely in the middle of the electromagnet.

(a) What current is needed in the coil ? ***Be sure to take fringing into account.*** [14]

(b) Determine the inductance of the coil. [3]

(c) What is the mass of the bar? [3]



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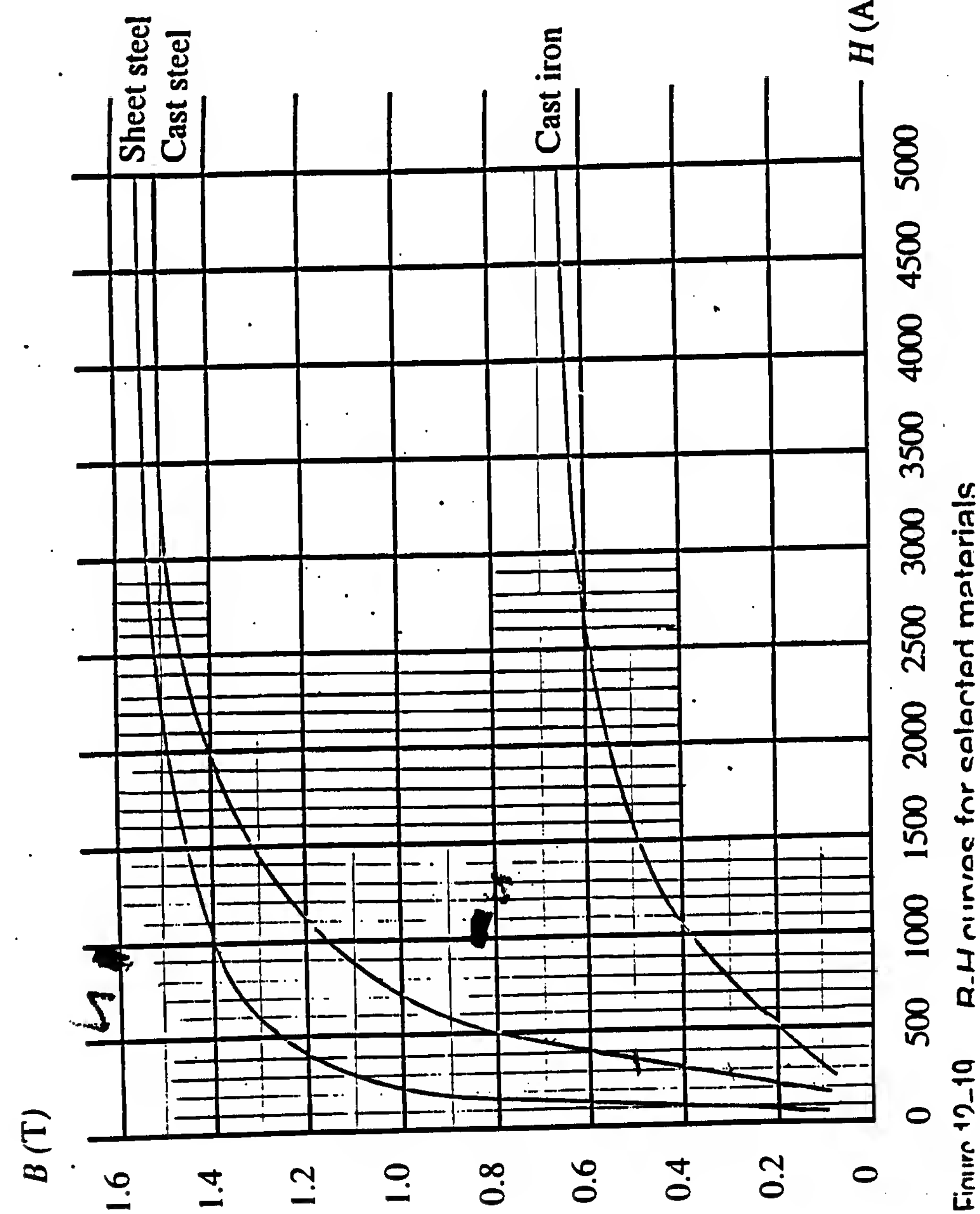


Figure 12-19 B-H curves for selected materials

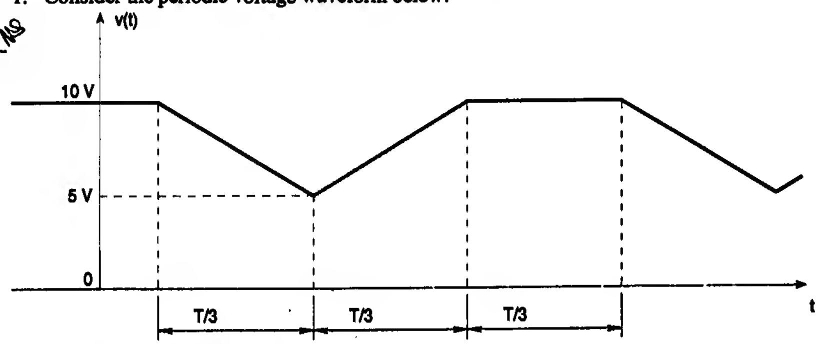
University of Saskatchewan
GE 212 Electric and Magnetic Circuits II
Final Examination

December 18, 1999
Allowed Time: 3 hours

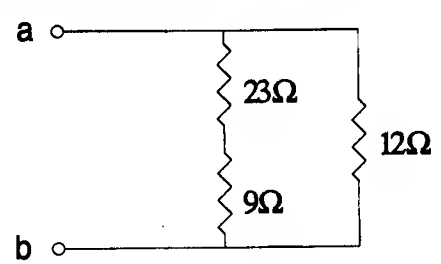
Closed Book Exam. Calculators and one 8.5" × 11" sheet of notes allowed.
All questions are of equal value. Complete all questions.

RMS

1. Consider the periodic voltage waveform below.

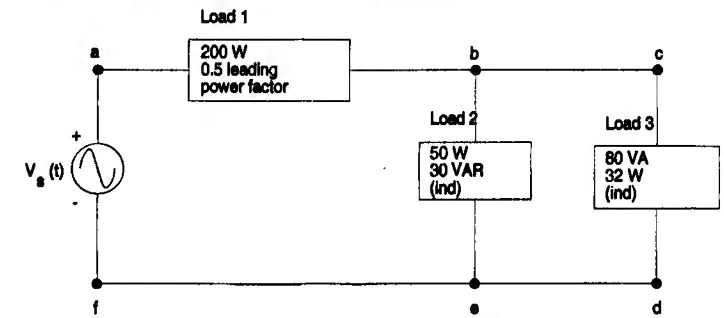


(a) What is the average value of the waveform? [4]
(b) What is the RMS value of the waveform? [6]
(c) What are the two possible readings if a D'Arsonval meter with a single series diode that was calibrated to read the RMS value of AC sinewaves was connected to the voltage source? [6]
(d) What would be the power dissipated by each resistor if the waveform were connected to the circuit below? [4]



2. In the circuit shown below, $v_s(t) = 100\sqrt{2}\sin(2\pi 60t + 10°)$ V :

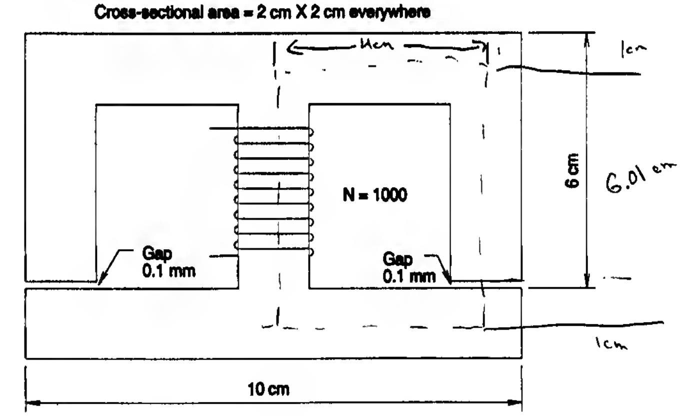
CP



(a) Draw the power triangle of each load and for the total load. [5]
(b) Determine the power factor of the circuit (as seen by the source) and the source current. [5]
(c) Determine the equivalent resistive and reactive element of the total load as seen by the source. [5]
(d) What element (type and magnitude) should be connected to the circuit to result in a power factor of 0.9 lagging (as seen by the source)? Where should this element be connected? (illustrate your answer) [5]

ME

3. A magnetic flux of 0.18 mWb is created in both gaps of a magnetic circuit, pictured below. The cast steel core is perfectly symmetrical with the center leg located precisely in the middle of the metal structure.

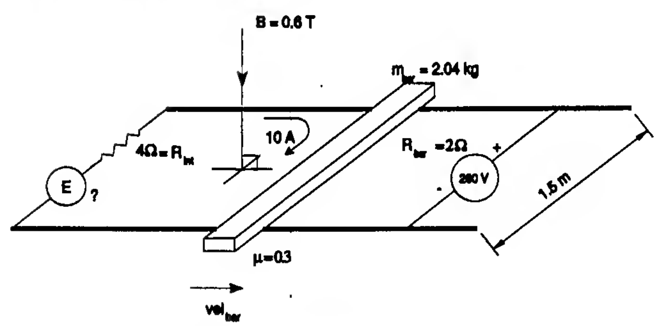


(a) Draw the electric equivalent circuit of the structure. [5]
(b) What current is required in the coil? *NEGLECT FRINGING!!!* [5]
(c) Determine the inductance of the coil. [5]
(d) Find the ratio of the reluctance of the left leg of the core to the reluctance of the right gap. [5]

B+R

4. The bar shown below has a mass of 2.04 kg and slides to the right with a ***constant*** velocity vel_{bar} on rails spaced 1.5 m apart. The rails have negligible resistance while the bar has an internal resistance of 2 Ω. The dynamic coefficient of friction is 0.3. There is a uniform perpendicular magnetic field, B = 0.6 T, acting downward throughout the region between the rails. The DC supply (internal resistance of 4 Ω) of unknown voltage and polarity together with the motional e.m.f. of the bar provide a current of 10 A flowing clockwise. Measurements show that the voltage difference between the rails is 260 V with the polarity as noted on the diagram.

(a) Draw the electric equivalent circuit. Indicate the polarity of the voltage source (E) and the motional e.m.f. (E_m). [3]
(b) Find the values of E and E_m. [4]
(c) Draw the free body diagram of the bar. [4]
(d) Calculate the magnitude of all forces acting on the bar. [3]
(e) Calculate the velocity of the bar, vel_{bar}. [2]
(f) Is the motor acting as a motor or a generator? (supply reason(s) for your answer) [2]
(g) Calculate the efficiency of the system. [2]

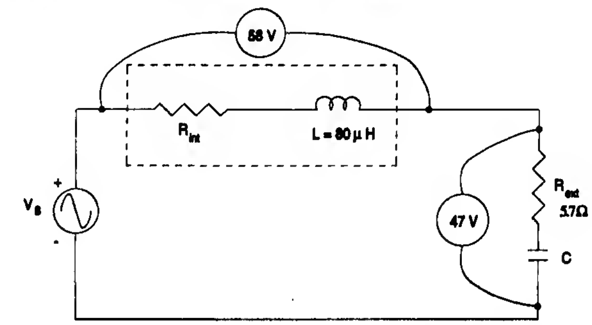


Res

5. A series circuit is constructed of a practical inductor with an unknown internal resistance R_{int}, an external resistor R_{ext} = 5.7 Ω, and a capacitor C. When the frequency of the voltage source is set to the resonant frequency of the circuit, the RMS voltage across the practical inductor is measured to be 58 V, while the RMS voltage across the external resistor and capacitance is measured to be 47 V. At

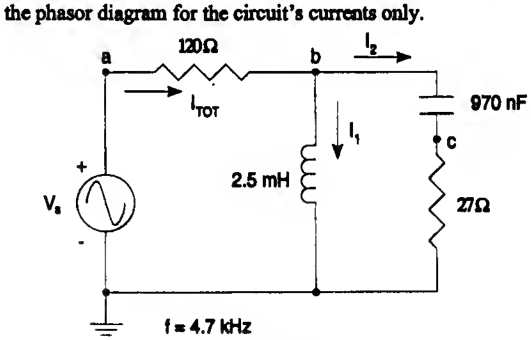
resonance, the phase difference between the voltage across the external resistor and the voltage across the capacitor is 59°. Find the following:

(a) The current I at resonance and R_{int}. [4]
(b) The resonant frequency f_s and the value of the capacitance C. [4]
(c) Derive an expression for the time-varying source voltage $v_s(t)$ = ? [4]
(d) What is the Q_{coil}? What is the Q of the circuit? [4]
(e) Calculate the ½ power frequencies and the bandwidth of the circuit. [4]



6. The frequency of the voltage source in the circuit below is f = 4.7 kHz, and V_s = 50∠35° V. Calculate the following:

(a) The impedance of the inductor and the impedance of the capacitor (magnitude and angle). [2]
(b) The total impedance seen by the source (magnitude and angle). [5]
(c) The currents I_{TOT}, I_1 and I_2 (express your answers in phasor form). [5]
(d) The voltage V_{ca} (express in phasor form). [5]
CA (e) Sketch the phasor diagram for the circuit's currents only. [3]



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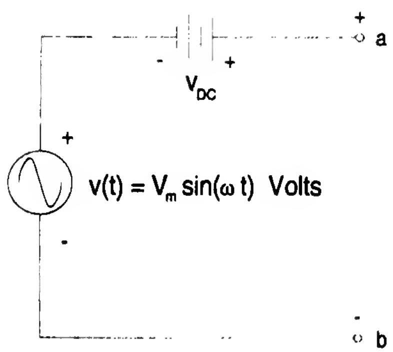
University of Saskatchewan
GE 212 Electric and Magnetic Circuits II
Final Examination

December 18, 1998
Allowed Time: 3 hours

Closed Book Exam. Calculators and one 8.5" × 11" sheet of notes allowed.
All questions are of equal value. Complete all questions.

RMS

1. [10] (a) Mathematically prove that for the following figure, the effective or RMS value of the voltage V_{ab} is $V_{ab} = \sqrt{V_{DC}^2 + V_{AC}^2}$, where $V_{AC} = V_m/\sqrt{2}$ is the effective value of the AC sinewave alone.

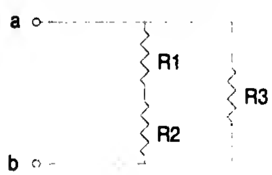


Average

[5] (b) What would a D'Arsonval meter that was calibrated to read DC voltages indicate if it were connected to V_{ab}? What assumptions, if any, concerning the frequency of the AC source must be made?

V_{DC}

[5] (c) What power would be dissipated by each of the resistors if the voltage source V_{ab} in part (a) above were connected to the following circuit?



$P_{R_3} = \dfrac{V_{DC}^2 + V_{AC}^2}{R_3}$

$P_{R_2} = \dfrac{V_{DC}^2 + V_{AC}^2}{(R_1+R_2)^2} (R_2)$

$P_{R_1} = \dfrac{V_{DC}^2 + V_{AC}^2}{(R_1+R_2)^2} (R_1)$

Power

2. Two loads are connected in series across a 120 V, 60 Hz supply. Load 1 draws 120 W and its' impedance $Z_1 = |Z_1|\angle 74.93°\,\Omega$; load 2 draws 258 VA at a power factor of 0.620 leading.

[10] (a) Draw the power triangle for the whole system and find the active, reactive and apparent powers of the total load. P = 280W Q = 243 VAR (ind) S = 371 VA

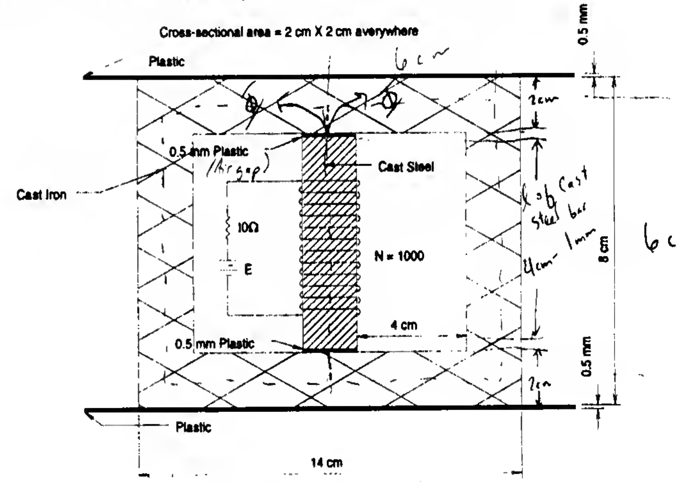
[4] (b) Find the total source current. Express your answer in phasor form. $I = 3.092\angle -40.9$

[6] (c) What reactive element is required to bring the total power factor of the load to unity? Where would this reactive element be connected? 104.4 μF

Magnetics

Pedo to check #'s

3. A child's toy is based on the principle of magnetic flux excitation in a rectangular core material made of cast iron, as shown below. The core is fixed between two plastic plates, each 0.5 mm thick. The magnetic flux is established by placing (exactly at the center of the core structure as shown) a 1000 turn coil wrapped on a cast steel bar. The bar is separated from the core by two 0.5 mm thick plastic plates. The cross-sectional area of both the cast iron core and cast steel bar is 2 cm × 2 cm. In order for the toy to run, a magnetic flux of at least 0.16 mWb must be created in the left leg of the core.



6 cm

6 cm 6 cm

[11] (a) What current in the coil is required to operate the toy? Be sure to take fringing into account. 0.836A

[3] (b) Will standard dry cell batteries be sufficient to run the toy? (1.5V) Yes

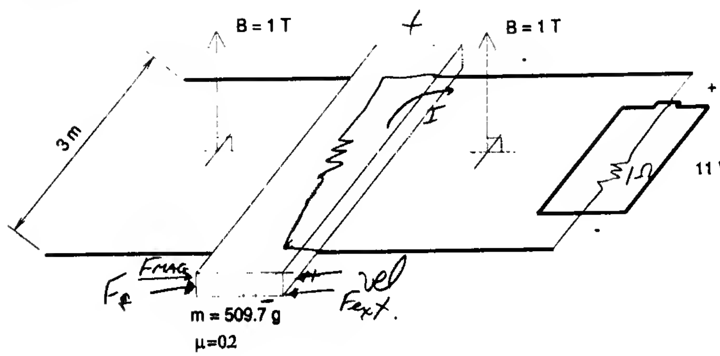
(continued)

[3] (c) Calculate the inductance of the coil. $l = \frac{NI}{I}$ 383 mH

[3] (d) What kind of gap material (specify) would significantly decrease the battery requirements? Ferromagnetic

4. In the bar and rail configuration below, an 11 Volt battery with an internal resistance of 1 Ohm is connected as illustrated. Every second, the bar does useful mechanical work of 6 J (i.e. it is acting as a motor); it has an electrical resistance of 1 Ohm and a mass of 509.7 g. The coefficient of static and kinetic friction between the bar and the rails is 0.2. A magnetic field is oriented up (perpendicular to the plane of the bar and rails) and has an intensity of 1 T.

6W



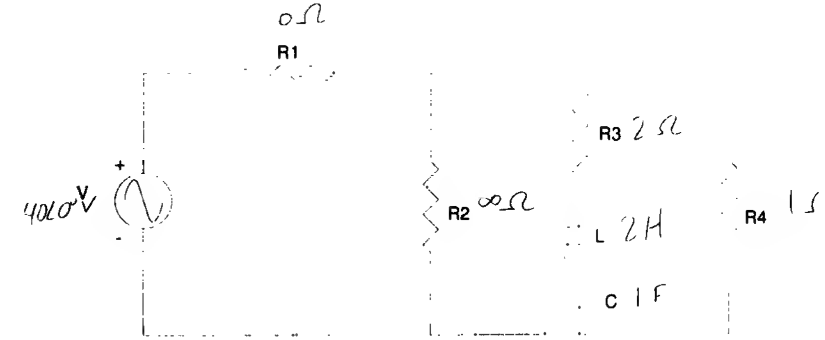
[1] (a) What is the direction of the bar's motion? ←

[10] (b) What is the current in the circuit (magnitude and direction), motional e.m.f. (polarity and magnitude) of the bar, and the voltage between the rails (polarity and magnitude)? 1, 4.83A ; 9V, 1.33V ; 10V, 6.163V

[6] (c) What is the magnitude and direction of the external force on the bar that is required to keep the bar moving at a constant velocity? 2N, 1.35N

[3] (d) This problem has two possible solutions for the bar's equilibrium velocity. Speculate how these two conditions could be achieved. 3, 4/9 m/s

5. In the circuit on the following page, the voltage source supplies a constant voltage $V = 40\angle 0°$ Volts. The frequency of the source is variable and is capable of being adjusted to any frequency. $R1 = 0\ \Omega$, $R2 = \infty\ \Omega$, $R3 = 2\ \Omega$, $R4 = 1\ \Omega$, $C = 1$ F, $L = 2$ H.



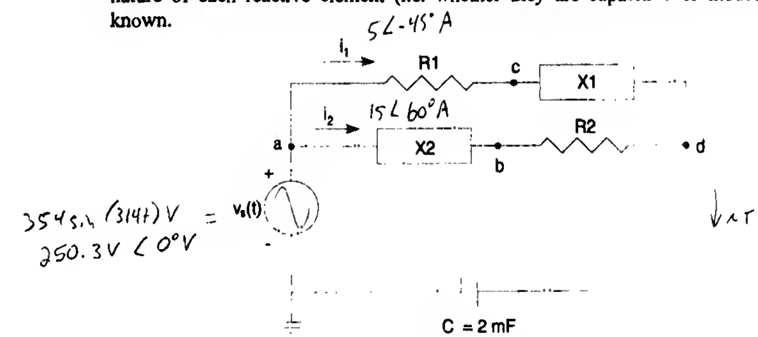
[5] (a) Calculate the resonant frequency of the circuit in Hz. 0.133Hz

[5] (b) Draw the equivalent circuit at resonance.

[5] (c) Find the current flowing through R4 at resonance. 40A

[5] (d) Calculate the voltage drops across C and L at resonance and sketch them on a phasor diagram. 28.28 V

6. In the circuit below, $i_1(t) = 7.07\sin(314t - \pi/4)$A and $i_2(t) = 21.2\sin(314t + \pi/3)$A. The supply voltage $v_s(t) = 354\sin(314t)$V. The two parallel impedances each consist of a resistor in series with a reactive element. The nature of each reactive element (i.e. whether they are capacitive or inductive) is not known.



[2] (a) Calculate the total supply current and express your answer as a time-varying sine function similar to $i_1(t)$ and $i_2(t)$ above. (continued)

$i_T(t) = 20.55 \sin(314t + 1.417 rad)$ A

University of Saskatchewan

Department of Mathematics & Statistics

MATH 223.3 – MIDTERM - TEST 1

October 15, 2001 Time: 80 minutes

Instructors: *(01) J. Brooke* *(03) B. Friberg* *(05) H. Teismann*

Note:

- **CLOSED BOOK – NO CALCULATORS**
- **ONE SHEET OF FORMULA ALLOWED**
- Questions 4, 5, 10, 11 are worth 1 mark each. All other questions are worth 2 marks each.
- Total of 20 marks.
- Properly encode your correct student number and intended answers on the blue opscan answer sheet.

Answer choices for all questions

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4
(F) 5	(G) 6	(H) 7	(I) 8	(J) 9

1. Consider the line through $(1,-2,-5)$ which is parallel to the line of intersection of the two planes $x+2y-z=5$ and $2x-y-z=3$. If $(a,b,0)$ is a point on this line, then $a+b=$
2. If D is the distance between the lines $x-1=\frac{y-1}{6}=\frac{z}{2}$ and $\frac{x-1}{2}=\frac{y-5}{15}=\frac{z+2}{6}$ then $7D-5=$
3. If D is the distance between the two planes $3x-2y+6z=10$ and $4y+6=6(x+2z)$, then $7D-1=$
4. Consider the parallelogram spanned by the vectors $\frac{1}{2}\langle -1,-6,7\rangle$ and $\frac{1}{2}\langle 5,-6,-1\rangle$. If D_1 and D_2 are the lengths of the longest and shortest diagonals, then $D_1-D_2-1=$
5. If $0\le\theta\le\frac{\pi}{2}$ is the angle between the two diagonals (in # 4), then $35\cos\theta=$
6. Let $(0,b,c)$ be the point where the tangent line to the curve $\vec{r}(t)=\langle e^t,t^3,\ln(t)\rangle$ at the point $(e,1,0)$ intersects the $y-z$ plane, then $b+c+10=$

7. The length of the curve $\vec{r}(t)=\langle t^2,2t,\ln(t)\rangle$ $1\le t\le e$ can be written as ae^2+be+c with a,b,c non-negative integers, and $a+2b+3c=$

8. If K is the curvature of the space curve $\vec{r}(t)=\langle t^2,t,2t\rangle$ at the point $(1,1,2)$ and $K=\frac{a+b\sqrt{5}}{27}$ with a and b non negative integers, then $a+b-1=$

9. The equation $x^2+y^2+z^2=4x-2y+6z-5$ describes a sphere of radius r centered at (a,b,c). Then $a+b+c+r=$

Questions 10–12 refer to the space curve $\vec{r}(t)=\left\langle \frac{t^2}{2}, \frac{(2t+1)^{3/2}}{3}, 0\right\rangle$ for $0\le t$.

10. If a_T is the tangential component of acceleration at the point $(8,9,0)$, then $10-6a_T=$

11. If a_N is the normal component of acceleration at the point $(8,9,0)$, then $10-6a_N=$

12. If s denotes the arc length parameter ($s=0$ when t=0), then $t=a+\sqrt{b+cs}$ with a,b,c integers and $a+2b+3c=$

** THE END **

Department of Mathematics & Statistics
MATH 223.3 – Midterm 2

November 18, 2002 — Time: 90 minutes

Instructors: *(01) E.D. Tymchatyn* *(03) A. Karassev* *(05) P. Browne*

Closed Book – No Calculators

Marks:

[2] 1. If $F(u,v,w) = u\cos(v+w)$ where $u = (x^3+1)y$, $v = xy$ and $w = e^{xy} + \frac{\pi}{4} - 1$, find $\frac{\partial F}{\partial x}$ at $(x,y) = (0,1)$.

[2] 2. Find the direction in which the function $f(x,y) = e^{2x+3y-25}$ increases most rapidly starting at the point $(3,7)$ and calculate the directional derivative in that direction.

[2] 3. Find the equation of the tangent plane to the surface $x = \sin(\pi yz)$ at $(0,1,5)$.

[3] 4. Find the absolute maximum and absolute minimum values of $f(x,y) = x^3 - 3x^2 + 6xy + 3y^2 - 9y$ in the rectangle with vertices $(0,0)$, $(2,0)$, $(0,1)$ and $(2,1)$.

[3] 5. Find the point on the sphere $x^2+y^2+z^2 = 4$ closest to the point $(4,3,5)$ by using the method of Lagrange multipliers.

[2] 6. Find $\iint_D \sin x^2 \, dA$ where D is the region bounded by $x = 0$, $x = \sqrt{\pi}$, $x = 2y$, $y = \frac{3x}{2}$.

[2] 7. Evaluate $\int_{-1}^{1}\int_{x^2}^{1} \frac{1}{1+y^{3/2}}\, dy\, dx$ by reversing the order of integration.

[2] 8. Using polar coordinates find $\iint_D (x^2+y^2)^{-1/2} dA$ where D is the region $(x-1)^2+y^2 \le 1$.

[2] 9. Find the centre of mass of a lamina which occupies the first quadrant of the circle $x^2+y^2 = 4$ and whose density at a point is proportional to distance from the origin.

[20] Total

$\boxed{90\%}$

** The End **

Department of Mathematics & Statistics
MATH 223.3 – Midterm 1

October 21, 2002 — Time: 80 minutes

Instructors: *(01) E.D. Tymchatyn* *(03) A. Karassev* *(05) P. Browne*

Closed Book – No Calculators

Show your work.

$\frac{21}{2} = \boxed{17}$

Marks:

[2] 1. Find the area of the triangle with vertices $P(3,0,1)$, $Q(1,1,1)$, $R(-2,1,0)$.

cross two vectors

[2] 2. Describe and sketch the surface $z = 4 - x^2$. parabola

[3] 3. Find the length of the curve $\vec{r}(t) = \langle e^{3t}\cos 3t,\ e^{3t}\sin 3t,\ e^{3t}\rangle$, $0 \le t \le 1$.

$A = \int_0^1 \sqrt{\vec{r}'(t)}$

[2] 4. Find the curvature of $y = \sin x$ at $x = \frac{\pi}{4}$.

$K = \frac{f''(x)}{(1+f'(x)^2)^{3/2}}$

[2] 5. Find the tangent plane to $z = x^2 + 3xy + y^2$ at the point where $(x,y) = (2,-2)$.

$z = 4 - 12 + 4 \quad z = -4$

[2] 6. Find the distance between the parallel planes

$$x+y+z+1 = 0$$
$$x+y+z+7 = 0\ .$$

[2] 7. Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$ or show it does not exist. (Squeeze theorm)

[2.5] 8. Find all unit vectors $\vec{u} = \langle a,b,c\rangle$ (i.e. $a^2+b^2+c^2 = 1$) so $\vec{u}$ makes an angle $\frac{\pi}{4}$ with each of the vectors $\vec{i}+\vec{j}$ and $\vec{i}+\vec{k}$.

[2.5] 9. Show that the curve $\vec{r}(t) = \langle t^2-t+1,\ t^3-t+2,\ \sin\pi t\rangle$ intersects itself at the point $P(1,2,0)$. Find the cosine of the angle of intersection at P.

[20] Total

** The End **

University of Saskatchewan
Department of Mathematics & Statistics
MATH 223.3 – Final Examination

Instructors: *(01) E. Tymchatyn* *(03) A. Karassev* *(05) P. Browne*

December 20, 2002 Time: 9:00–12:00

CLOSED BOOK – NO CALCULATORS

[6] ?? Find the point on the line through $P(1,1,1)$ and $Q(2,3,4)$ which is closest to the origin.

[6] Identify and sketch the surface whose equation in spherical coordinates is $\rho^2(\sin^2\phi\cos^2\theta+\cos^2\phi)=4$.

[7] Show that the lines
$L_1: \quad x-1=\frac{y+6}{2}=z-2$ and $L_2: \quad \frac{x}{2}=y-4=\frac{z-1}{2}$
intersect. Find the equations of the line passing through the point of intersection of L_1 and L_2 and which is perpendicular to both L_1 and L_2.

[5] For the curve $\mathbf{r}(t)=\langle \frac{t^3}{3},\frac{t^2}{2},t\rangle$ find
(a) the unit tangent vector. $T(t)=\frac{r'(t)}{|r'(t)|}$
(b) the curvature. $\kappa=\frac{|r'(t)\times r''(t)|}{|r'(t)|}$

[5] Find $\lim_{(x,y)\to(0,0)} x^3\cot(x^2+y^2)$ or show it does not exist.

[5] Use differentials to find an approximate value for $f(1.9,2.01)$ where $f(x,y)=\sqrt{x^3y}$.
dz
$dz = f_x(x-x_0)+f_y(y-y_0)$

[5] A particle moves in space so its position at time t is given by $\mathbf{r}(t)=\langle t^2,5t,t^2-16t\rangle$. When is its speed a minimum?

[5] The length x of a side of a triangle is increasing at 3 cm/sec. The length y of another side is decreasing at 2 cm/sec and the included angle θ is decreasing at .1 radians/sec. How fast is the area of the triangle changing when $x=20$, $y=30$ and $\theta=\frac{\pi}{6}$?
Chain Rule

.../2

35=??

[7] Let $P(x_0,y_0,z_0)$ be any point on the surface ($x_0\neq 0,\ y_0\neq 0,\ z_0\neq 0$)
$x^{1/2}+y^{1/2}+z^{1/2}=a^{1/2}$, ($a>0$, a constant).
Find:
(i) the tangent plane at P. $f_x(x-x_0)+f_y(y-y_0)+f_z(z-z_0)$
(ii) the points at which the tangent plane intersects the three coordinate axes.
(iii) the sum of these three intercepts.

[8] Maximize $f(x,y,z)=xyz$ subject to $x^3+y^3+z^3=1$.
lagrange / constraint

[9] (a) Show that the sphere $x^2+y^2+z^2=2$ and the paraboloid $z=x^2+y^2$ intersect in a circle lying on the plane $z=1$.
(b) Find the volume of the region inside the sphere $x^2+y^2+z^2=2$ and above the paraboloid $z=x^2+y^2$.

[5] ?? Find the surface area of the portion of the cone $z^2=x^2+y^2$ that lies between the planes $z=0$ and $z=1$.

[5] Find $\iiint_B z\,dV$ where B is the top half of the ball $x^2+y^2+z^2=1$.

[5] Use a change of variable to compute $\iint_D (x+y)^2\,dA$ where D is the paralleligram bounded by $x+y=0$, $x+y=1$, $2x-y=0$, $2x-y=3$.

[5] ???. Find $\int_C \mathbf{F}\cdot d\mathbf{r}$ if $\mathbf{F}=\langle x,x,x\rangle$ and C is the helix $\mathbf{r}(t)=\langle\cos t,\sin t,t\rangle$, $0\le t\le 2\pi$.
$\int_a^b f(r(t))|r'(t)|\,dt$

[5] Show that the line integral $\int_C (2y^2-12x^3y^3)dx+(4xy-9x^4y^2)dy$ is independent of path and evaluate the integral where C is any piecewise smooth path from $(1,1)$ to $(3,2)$.

[7] Evaluate $\int_C x\sqrt{y}\,dx+2y\sqrt{x}\,dy$ where C consists of the counterclockwise path on the circle $x^2+y^2=1$ from $(1,0)$ to $(0,1)$ followed by the line segment from $(0,1)$ to $(4,3)$.

[0] Total

THE END

University of Saskatchewan

Department of Mathematics & Statistics

MATH 223.3 - MIDTERM - TEST 2

November 26, 2001 | Time: 80 minutes

Instructors: *(01) J. Brooke* *(03) B. Friberg* *(05) H. Teismann*

Note:

- **CLOSED BOOK – NO CALCULATORS**
- ONE SHEET OF FORMULA ALLOWED
- Properly encode your correct student number and intended answers on the blue opscan answer sheet.

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1. The largest value of the directional derivative for the function $f(x,y) = x^2y - 2y^2$ at the point $(2,1)$ is:

   (A) 2 (B) $\frac{5}{2}$ (C) 3 (D) $\frac{7}{2}$ (E) 4

   (F) $\frac{9}{2}$ (G) 5 (H) $\frac{11}{2}$ (I) 6 (J) $\frac{13}{2}$

2. In which direction $\theta (0 \leq \theta \leq \pi)$ measured counter clockwise from the positive x-axis is the directional derivative for the function (in #1) equal to -2?

   (A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$ (E) $\frac{\pi}{2}$

   (F) $\frac{2\pi}{3}$ (G) $\frac{3\pi}{4}$ (H) $\frac{5\pi}{6}$ (I) π

3. The maximum value of $f(x,y) = x + y$ subject to the constraint $2x^2 + y^2 = 1$ is

   (A) 1 (B) $\frac{\sqrt{6}}{2}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$ (E) $\sqrt{6}$

   (F) $\frac{1+\sqrt{2}}{2}$ (G) $\frac{1+\sqrt{3}}{2}$ (H) $\frac{1+\sqrt{6}}{2}$ (I) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (J) 2

.../2

---

4. Let $L(x,y)$ be the tangent plane approximation (linearization) to the function $f(x,y) = xe^{xy}$ at the point $(2,1)$, then $L(\frac{5}{2}, 0) =$

   (A) $-\frac{5e^2}{2}$ (B) $-2e^2$ (C) $-\frac{3e^2}{2}$ (D) $-e^2$ (E) $-\frac{e^2}{2}$

   (F) 0 (G) $\frac{e^2}{2}$ (H) e^2 (I) $\frac{3e^2}{2}$ (J) $2e^2$

5. $z^2 + \frac{2xz}{y} = 6$ defines z implicity as a function of x and y near the point $P = (1,2,-3)$. What is $\frac{\partial z}{\partial x}$ at this point?

   (A) -1 (B) $-\frac{4}{5}$ (C) $-\frac{3}{5}$ (D) $-\frac{2}{5}$ (E) $-\frac{1}{5}$

   (F) 0 (G) $\frac{1}{5}$ (H) $\frac{2}{5}$ (I) $\frac{3}{5}$ (J) $\frac{4}{5}$

6. The tangent plane to the graph of $z = f(x,y) = \frac{x^2}{x+y}$ at the point $(1,1,\frac{1}{2})$ intersects the z-axis in a point whose distance from the origin $(0,0,0)$ is

   (A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

   (F) $\frac{3}{4}$ (G) 1 (H) $\frac{5}{4}$ (I) $\frac{4}{3}$ (J) $\frac{3}{2}$

7. The function $f(x,y) = x^3 + y^2 - xy$ has two critical points; at P and at $Q = (0,0)$. Which of the following describes P and Q?

   (A) local minimum at P and local minimum at Q
   (B) local minimum at P and local maximum at Q
   (C) local minimum at P and saddle at Q
   (D) local maximum at P and local minimum at Q
   (E) local maximum at P and local maximum at Q
   (F) local maximum at P and saddle at Q
   (G) saddle at P and local minimum at Q
   (H) saddle at P and local maximum at Q
   (G) saddle at P and saddle at Q

.../3

8. Let D be the semi-~~circle~~ Disc $x^2+y^2 \leq 1$; $0 \leq y$. Then $\iint\limits_D (x^2y+\sin x)dA =$

(A) 0 (B) $\frac{1}{15}$ (C) $\frac{1}{10}$ (D) $\frac{2}{15}$ (E) $\frac{1}{5}$

(F) $\frac{4}{15}$ (G) $\frac{3}{10}$ (H) $\frac{1}{3}$ (I) $\frac{2}{5}$ (J) $\frac{1}{2}$

9. Evaluate $\int_0^1 dx \int_x^1 e^{y^2}dy$ by reversing the order of iteration.

(A) $\frac{e+2}{2}$ (B) $\frac{e+1}{2}$ (C) $\frac{e}{2}$ (D) $\frac{e-1}{2}$ (E) $\frac{e-2}{2}$

(F) $e+2$ (G) $e+1$ (H) e (I) $e-1$ (J) $e-2$

10. Consider the two regions D_1 and D_2 defined by

$D_1 : 0 \leq x \leq 1$ and $0 \leq y \leq \sqrt{x}$

$D_2 : 1 \leq x$ and $0 \leq y \leq \sqrt{x}$

Let $I_1 = \iint\limits_{D_1} \frac{y}{x^3}\,dA$ and $I_2 = \iint\limits_{D_2} \frac{y}{x^3}\,dA$.

Choose the correct statement regarding I_1 and I_2 .

(A) Both I_1 and I_2 diverge

(B) Both I_1 and I_2 converge

(C) I_1 diverges and I_2 converges to $\frac{1}{4}$

(D) I_1 diverges and I_2 converges to $\frac{1}{2}$

(E) I_1 diverges and I_2 converges to $\frac{3}{4}$

(F) I_1 diverges and I_2 converges to 1

(G) I_2 diverges and I_1 converges to $\frac{1}{4}$

(H) I_2 diverges and I_1 converges to $\frac{1}{2}$

(I) I_2 diverges and I_1 converges to $\frac{3}{4}$

(J) I_2 diverges and I_1 converges to 1

** THE END **

University of Saskatchewan
Department of Mathematics & Statistics
MATH 223.3 – Final Examination

Instructors: *(01) J. Brooke* *(03) B. Friberg* *(05) H. Teismann*

December 18, 2001 Time: 9:00–12:00

- CLOSED BOOK – NO CALCULATORS
- TWO SHEETS OF FORMULA ALLOWED
- 20 questions of equal value.
- Properly encode your correct student number and intended answers on the blue opscan answer sheet.

Answer choices for questions 1 – 17 are:

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4
(F) 5	(G) 6	(H) 7	(I) 8	(J) 9

1. Consider the line through $(2,3,-1)$ which intersects neither of the planes $2x-y+z=5$ and $3x+y-z=5$. If $(a,2,c)$ is a point on this line then $a+c=$.

2. The distance from the plane $4x-y+z=4$ to the origin can be written as $\frac{a+b\sqrt{2}}{3}$ with a and b integers and $a+b=$.

3. The tangent line to the curve $\vec{r}(t)=(\ln(t),t^2,e^{2t})$ at the point $(0,1,e^2)$ intersects the plane $z=3e^2$ in a point whose first coordinate is .

4. The length of the curve $\vec{r}(t)=(t,\sqrt{8}t,\ln(t))$, $1\le t\le e$, can be written as ae^2+be+c with a,b, and c integers and $a+b+c=$.

5. The radius of curvature which remains constant along the spiral $\vec{r}(t)=(\cos t,\sin t,t\sqrt{8})$ is equal to .

6. If s denotes the arc length parameter ($s=0$ when $t=0$) for the curve $\vec{r}(t)=(2(2t+1)^{\frac{3}{2}},3t^2)$, $0\le t$, then $t=-a+\sqrt{a+\frac{s}{b}}$ with a and b integers and $a+b=$.

7. The minimum value of the function $f(x,y)=3+xy$ subject to the constraint $x^2+y^2=2y$ can be written as $\frac{a+b\sqrt{3}}{4}$, with a and b integers and $a+b=$.

8. $z^2+\frac{xz}{y}+2=0$ defines z implicitly as a function of x and y near the point $P=(3,-1,2)$. What is $\frac{\partial z}{\partial y}$ at this point?

9. Let $L(x,y)$ be the tangent plane approximation (linearization) to the function $f(x,y)=xye^{xy}$ at the point $(2,0)$. Then $L(\ln(2),4)$ is .

10. Consider the normal line to the graph of $z=xy$ at the point $(1,2,2)$. Its distance from the origin can be written as $\sqrt{\frac{20+a}{3}}$ where $a=$.

11. Let D be the elliptical disk $4x^2+y^2\le 4$. Then $\iint_D (y+x^2\sin(x^3))\,dA=\frac{a\pi}{9}$ where $a=$.

12. $\int_0^1 dy\int_{y^2}^1 4ye^{x^2}\,dx = ae+b$ where a and b are integers and $a+b=$.
(Hint: Reverse the order of iteration.)

13. If C is the semi-circle $x^2+y^2=1$ and $y\ge 0$ then $\int_C x^2y\,ds=\frac{a}{9}$ where $a=$.

14. Consider the conservative vector field $\vec{F}(x,y,z)=(y^2+z^3,2xy,3xz^2)$. Let ϕ be a potential function for $\vec{F}$ with $\phi(0,0,0)=0$. Then $\phi(3,2,-1)=$.

15. Let C be the curve $\vec{r}(t)=(\sin\left(\frac{t^2\pi}{2}\right),e^{t^2})$ $0\le t\le 1$. Then $\int_C(6xy^2\,dx+6x^2y\,dy)=\int_C \vec{F}\cdot d\vec{r}$ with $\vec{F}(x,y)=(6xy^2,6x^2y)$ valuates to ae^2+be+c with a,b, and c integers and $3a+2b+c=$.

16. If B is the unit ball $x^2+y^2+z^2\le 1$ then $\iiint_B \sqrt{x^2+y^2+z^2}\,dV=\frac{a\pi}{6}$ where $a=$.
(Hint: Spherical coordinates works nicely.)

17. Consider the region D defined by $1 \le x^2 + y^2 \le 4;\ \ x \ge 0;\ \ y \ge 0$.
Then $\displaystyle\iint_D x\,dA = \frac{a}{3}$ where $a =$ ______.

18. In which direction $\theta \left(-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right)$ measured from the positive x-axis is the directional derivative for the function $f(x,y) = xe^{xy}$ at the point $(2,1)$ equal to zero?

(A) $\arctan\left(\frac{5}{6}\right)$ (B) $\arctan\left(\frac{4}{5}\right)$ (C) $\arctan\left(\frac{3}{4}\right)$ (D) $\arctan\left(\frac{2}{3}\right)$

(E) $\arctan\left(\frac{1}{2}\right)$ (F) $\arctan\left(-\frac{1}{2}\right)$ (G) $\arctan\left(-\frac{2}{3}\right)$ (H) $\arctan\left(-\frac{3}{4}\right)$

(I) $\arctan\left(-\frac{4}{5}\right)$ (J) $\arctan\left(-\frac{5}{6}\right)$

19. Choose the statement which describes all the critical points of $f(x,y) = x^2 - y^2 + x^2y$.

(A) f has only one critical point; a saddle point.

(B) f has only one critical point; a local minimum.

(C) f has only one critical point; a local maximum.

(D) f has exactly two critical points; a saddle and local minimum.

(E) f has exactly two critical points; a saddle and local maximum.

(F) f has exactly two critical points; a local minimum and local maximum.

(G) f has exactly two critical points; both saddle points.

(H) f has exactly two critical points; both local minima.

(I) f has exactly two critical points; both local maxima.

(J) f has more than two critical points.

.../4

20. Consider the two regions D_1 and D_2 defined by

$$D_1 : 0 \le x \le 1 \quad \text{and} \quad 0 \le y \le x^{\frac{1}{3}}$$

$$D_2 : 1 \le x \quad \text{and} \quad 0 \le y \le x^{\frac{1}{3}}.$$

Let $I_1 = \displaystyle\iint_{D_1} \frac{y^2}{x}\,dA$ and $I_2 = \displaystyle\iint_{D_2} \frac{y^2}{x}\,dA$.
Choose the correct statement regarding I_1 and I_2.

(A) Both I_1 and I_2 diverge.

(B) Both I_1 and I_2 converge.

(C) I_1 diverges and I_2 converges to $\frac{1}{4}$.

(D) I_1 diverges and I_2 converges to $\frac{1}{3}$

(E) I_1 diverges and I_2 converges to $\frac{1}{2}$

(F) I_1 diverges and I_2 converges to 1

(G) I_2 diverges and I_1 converges to $\frac{1}{4}$

(H) I_2 diverges and I_1 converges to $\frac{1}{3}$

(I) I_2 diverges and I_1 converges to $\frac{1}{2}$

(J) I_2 diverges and I_1 converges to 1

** THE END **

University of Saskatchewan

Department of Mathematics & Statistics
MATH 223.3 – MIDTERM - TEST 1

Instructors: *(01) P. J. Browne* *(03) B. Friberg* *(05) H. Teismann*

October 16, 2000 Time: 4:30–6:00 pm

Note:
- This is an open book exam. Students may use Stewart: Calculus.
- Calculators, notes, formula sheets are not permitted.
- Only a soft lead pencil should be used on the opscan sheet.
- Your student ID number should be encoded and written in the upper left hand corner of the opscan sheet.

ALL QUESTIONS ARE OF EQUAL VALUE

Section A:
Answer these questions on the opscan sheet.

1. Describe the set of all points $P(x,y,z)$ so that the distance from P to $(3,0,-3)$ is twice the distance from P to $(0,0,0)$.

(A) Sphere: centre $(1,0,-1)$ radius $\sqrt{8}$.
(B) Sphere: centre $(-1,0,1)$, radius $\sqrt{8}$
(C) Sphere: centre $(-1,0,-1)$, radius $\sqrt{8}$
(D) Ellipsoid: $\frac{(x+1)^2}{4}+y^2+\frac{(z-1)^2}{8}=1$
(E) Sphere: centre $(-1,0,1)$, radius 8
(F) Sphere: centre $(-1,0,1)$, radius $\sqrt{6}$
(G) Sphere: centre $(1,0,-1)$, radius $\sqrt{6}$
(H) Sphere: centre $(-1,0,1)$, radius 2
(I) None of the above.

2. Find the line through $(1,1,1)$ which is parallel to the line of intersection of the two planes $3x+2y+z+1=0$ and $x+y+z+2=0$.

(A) $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-1}{1}$
(B) $\frac{x+1}{1}=\frac{y+1}{1}=\frac{z+1}{1}$
(C) $\frac{x+1}{1}=\frac{y+1}{-2}=\frac{z+1}{1}$
(D) $\frac{x+1}{1}=\frac{y+2}{1}=\frac{z+1}{1}$
(E) $\frac{x-1}{1}=\frac{y-1}{-2}=\frac{z-1}{1}$
(F) $\frac{x-1}{1}=\frac{y+2}{1}=\frac{z-1}{1}$
(G) $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-1}{1}$
(H) $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-1}{1}$
(I) None of the above.

/...2

3. Find the plane containing the points $(1,1,1)$ and $(-2,0,3)$ and which is not intersected by the line $x=-4+t$, $y=2$, $z=3-t$.

(A) $x-y+z-1=0$
(B) $x+5y-z+5=0$
(C) $2x-4y+z+1=0$
(D) $2x-8y-z+7=0$
(E) $3x-7y+z+3=0$
(F) $3x-11y-z+9=0$
(G) $4x-10y+z+5=0$
(H) $2x+4y+z-7=0$
(I) None of the above.

4. Find the distance between the two parallel planes $2x+3y-z+4=0$ and $2x+3y-z+18=0$.

(A) 22 (B) $1/\sqrt{14}$ (C) $\sqrt{14}$ (D) 14 (E) $22/\sqrt{14}$
(F) 1 (G) $14/\sqrt{6}$ (H) $22/\sqrt{6}$ (I) None of the above.

5. Find the distance between the two parallel lines $x-2=y+3=z-4$ and $x=y=z$.

(A) $\sqrt{2}$ (B) 2 (C) $\sqrt{8}$ (D) 8
(E) $\sqrt{42}$ (F) 42 (G) 26 (H) $\sqrt{26}$
(I) None of the above.

6. The curve $9y=x^2$ in the X-Y plane is rotated about the Y-axis. What is the quadric surface generated and what is its equation?

(A) Circular paraboloid; $z=\frac{x^2}{9}+\frac{y^2}{9}$
(B) Circular paraboloid; $9y=x^2+z^2$
(C) Elliptic paraboloid; $9y=x^2+9z^2$
(D) Ellipsoid; $\frac{x^2}{9}+\frac{z^2}{9}+y^2=1$
(E) Hyperboloid of one sheet; $9x^2+9z^2-y^2=1$
(F) Hyperboloid of one sheet; $\frac{x^2}{9}+\frac{z^2}{9}-y^2=1$
(G) Hyperbolic cylinder; $9y^2=x^2$
(H) Cone: $x^2+y^2=81z^2$
(I) None of the above.

/...3

7. Find the tangent line to the curve $\underset{\sim}{r}(s) = e^{-2s}\underset{\sim}{i} + \cos s\,\underset{\sim}{j} + 3\sin s\,\underset{\sim}{k}$ at the point $(1,1,0)$ on this curve. Where does this tangent line intersect the YZ plane?

(A) $(0,1,\frac{3}{2})$ (B) $(0,0,\frac{3}{2})$ (C) $(0,1,-\frac{3}{2})$ (D) $(0,0,-\frac{3}{2})$ (E) $(0,\frac{1}{2},\frac{3}{2})$
(F) $(0,1,3)$ (G) $(0,0,3)$ (H) $(0,0,-3)$ (I) None of the above.

8. Find the arc length of the curve $\underset{\sim}{r}(t) = < t^3, t, \frac{1}{2}\sqrt{6}t^2 >,\ 1 \le t \le 3$.

(A) 30 (B) $\sqrt{26}$ (C) 26 (D) 4 (E) $\sqrt{28}$
(F) $\sqrt{30}$ (G) 28 (H) 2 (I) None of the above.

9. What is the maximum value of the curvature for the function $y = \ln\cos x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$.

(A) $\sqrt{2}$ (B) $1/\sqrt{2}$ (C) 1 (D) $\ln 2$ (E) ∞
(F) $\sqrt{3}/2$ (G) 0 (H) 1/2 (I) None of the above.

10. A point P has rectangular co-ordinates $(1,-1,\sqrt{2})$. What are its spherical co-ordinates (ρ,θ,ϕ)?

(A) $\left(2,\frac{7\pi}{4},\frac{\pi}{4}\right)$ (B) $\cancel{\left(2,\frac{\pi}{4},\frac{\pi}{4}\right)}$ (C) $\left(2,\frac{7\pi}{4},\frac{3\pi}{4}\right)$ (D) $\cancel{\left(2,\frac{\pi}{4},\frac{3\pi}{4}\right)}$
(E) $\left(2,\frac{5\pi}{4},\frac{\pi}{4}\right)$ (F) $\cancel{\left(2,\frac{5\pi}{4},\frac{3\pi}{4}\right)}$ (G) $\left(2,\frac{3\pi}{4},\frac{\pi}{4}\right)$ (H) $\left(2.\frac{3\pi}{4},\frac{3\pi}{4}\right)$
(I) None of the above.

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BONUS QUESTION

Write a detailed solution to this question in the answer booklet provided.

B1. A surface has equation in spherical co-ordinates

$$\rho\sin\phi = 2\cos\theta\ .$$

Express this equation in rectangular co-ordinates, identify and sketch.

** THE END **

University of Saskatchewan

Department of Mathematics & Statistics
MATH 223.3 – MIDTERM - TEST 2

Instructors: *(01) P. J. Browne* *(03) B. Friberg* *(05) H. Teismann*

November 13, 2000 Time: 4:30–6:00 pm

Note:
- This is an open book exam. Students may use Stewart: Calculus.
- Calculators, notes, formula sheets, etc. are not permitted.
- Only a soft lead pencil should be used on the opscan sheet.
- Your student ID number should be encoded and written in the upper left hand corner of the opscan sheet. Also, **PRINT** your name.

ALL QUESTIONS ARE OF EQUAL VALUE

Section A: Answer these questions on the opscan sheet

1. Find $\lim\limits_{(x,y)\to(0,0)} \left(\dfrac{2x^4+3y^4}{|x|^3+|y|^3}\right)$

(A) 2 (B) 3/2 (C) 2/3 (D) ∞ (E) 0
(F) 3 (G) 1 (H) Does not exist (I) None of the above.

2. Find the tangent plane to the surface $z = 3x^2 + 4y^2 - 6xy$ at the point on the surface where $x = 1,\ y = 1$.

(A) $2y + z + 3 = 0$ (B) $2y + z - 3 = 0$ (C) $x + 2y + z - 4 = 0$
(D) $x = 1$ (E) $x + y + z + 3 = 0$ (F) $2y - z - 1 = 0$
(G) $x + y + z - 3 = 0$ (H) $2y - z + 1 = 0$ (I) None of the above

6

3. Let $f(x,y,z) = \sqrt{x^2+y^2+z^2}$. Use differentials based at $(3,2,\cancel{7})$ to find an approximate value for $f(2.9, 1.9, 6.2)$.

(A) 7.10 (B) 7.20 (C) 6.90 (D) 7.24 (E) 7.01
(F) 7.02 (G) 6.09 (H) 7.024 (I) None of the above.

4. The radius of a right circular cylinder is decreasing at the rate of 2 cms/sec and the height is increasing at the rate of 3 cms/sec. At what rate is the volume of the cylinder changing at the instant the radius is 13 cms and the height is 18 cms?

(A) 429π (B) 1043π (C) -429π (D) -1043π (E) -936π

(F) 507π (G) -143π (H) 143π (I) None of the above.

5. If $t = z\sec(xy)$, $x = uv$, $y = vw$, $z = wu$, find $\dfrac{\partial t}{\partial w}$ when $u = \dfrac{\pi}{4}$, $v = \dfrac{\pi}{4}$, $w = 0$.

(A) $\dfrac{\pi^2}{16} - \dfrac{\pi}{4}$ (B) $\dfrac{-\pi^2}{16}$ (C) $\dfrac{-\pi}{4}$ (D) 0 (E) Undefined

(F) $\dfrac{\pi^2}{16} + \dfrac{\pi}{4}$ (G) $\dfrac{\pi}{4}$ (H) $\dfrac{\pi^2}{16}$ (I) None of the above.

6. Find and classify the stationary (critical) points of $f(x,y) = xy + \dfrac{1}{x} + \dfrac{1}{y}$

(A) $(0,1)$, $(1,0)$ local maxima

(B) $(0,1)$, $(1,0)$ saddles, $(1,1)$ local min

(C) $(0,1)$, $(1,0)$ local maxima, $(1,1)$ local min

(D) $(0,1)$, $(1,0)$ local minima

(E) $(0,1)$, $(1,0)$, $(1,1)$ local minima

(F) $(1,1)$ saddle

(G) $(1,1)$ local min

(H) $(1,1)$ local max

(I) None of the above.

7. Find the directional derivative of $f(x,y,z) = x^2 - 2y^2 + 2z^2$ at the point $(1,2,3)$ in the direction of the origin

(A) -22 (B) 22 (C) $\dfrac{7}{11}$ (D) $\dfrac{\sqrt{14}}{22}$ (E) $\dfrac{-22}{\sqrt{14}}$

(F) $\dfrac{22}{\sqrt{14}}$ (G) $\dfrac{11}{7}$ (H) $\dfrac{-11}{7}$ (I) None of the above.

/...3

8. Find $\int\int_R y(x^2+y^2)^3 dA$ where R is the region bounded by the X-axis and the lines $y = -x$, $x = 1$.

(A) $-5/12$ (B) $5/12$ (C) $-15/56$ (D) $-5/24$ (E) $5/24$

(F) $15/56$ (G) $-15/24$ (H) $15/24$ (I) None of the above.

9. Find the volume of the tetrahedron in the first octant bounded by the co-ordinate planes and the plane $2x + 2y + z = 4$

(A) $8/3$ (B) $16/3$ (C) $4/3$ (D) $2/3$ (E) 8

(F) 2 (G) 16 (H) 4 (I) None of the above.

10. Evaluate $\displaystyle\int_0^1 \int_y^{\sqrt{1-y^2}} \sqrt{x^2+y^2}\, dx\, dy$

(A) $\dfrac{1}{12}$ (B) $\pi/12$ (C) $\pi/8$ (D) $\dfrac{1}{8}$

(E) $\pi/16$ (F) $\pi/24$ (G) $\dfrac{1}{24}$ (H) $\dfrac{1}{16}$

(I) None of the above.

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Section B: Answer this question in the booklet provided.

B1. Find the minimum value of $f(x,y,z) = x + y + z$ subject to $x^4 + y^4 + z^4 = 48$. What point(s) gives this minimum?

** THE END **

University of Saskatchewan

Department of Mathematics & Statistics
MATH 223.3 – MIDTERM - EXAMINATION 1

Instructors: *(01) H. S. Yin* *(03) J. A. Brooke* *(05) P. J. Browne*

October 18, 1999 Time: 4:30–6:00 pm

Note:
• This is an open book exam. Students may use Stewart: Calculus.
Calculators, notes, formula sheets are not permitted.
• Only a soft lead pencil should be used on the opscan sheet.
• Your student ID number should be encoded and written in the upper left hand corner of the opscan sheet.

Section A: Each Question is worth 3 marks

1. The set of points (x, y, z) equidistant from $(1,2,3)$ and $(3,6,-1)$ has equation:

(A) $x+2y-2z+10=0$ (B) $x+2y-2z-10=0$
(C) $x+2y-2z+20=0$ (D) $x+2y-2z-20=0$
(E) $x+2y-2z-8=0$ (F) $x+2y+2z-10=0$
(G) $x+2y+2z-8=0$ (H) $x+2y-2z-16=0$
(I) None of the above.

2. The line through the point $(2,5,6)$ and parallel to the line through $(2,4,7)$ and $(3,0,1)$ has equations:

(A) $\frac{x-2}{1}=\frac{y-5}{-4}=\frac{z-6}{-6}$ (B) $\frac{x-2}{1}=\frac{y-5}{4}=\frac{z-6}{6}$
(C) $\frac{x-1}{2}=\frac{y+4}{5}=\frac{z+6}{6}$ (D) $\frac{x-1}{2}=\frac{y+4}{-5}=\frac{z+6}{-6}$
(E) $\frac{x-3}{2}=\frac{y}{4}=\frac{z-1}{7}$ (F) $\frac{x-2}{3}=\frac{y}{5}=\frac{z-6}{1}$
(G) $\frac{x-2}{2}=\frac{y-4}{5}=\frac{z-7}{6}$ (H) $x=2, \frac{y-5}{4}=\frac{z-6}{7}$
(I) None of the above.

/...2

3. The equation of the plane through $(2,1,0)$ and containing the line $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z}{3}$ is

(A) $2x+y+3z-5=0$ (B) $-x+y+1=0$
(C) $x+y+z-3=0$ (D) $3y+z-3=0$
(E) $2x+y+3z+5=0$ (F) $3y-z-3=0$
(G) $-x+y-1=0$ (H) $x+y+3z-3=0$
(I) None of the above.

4. Find the distance between the two parallel planes $x+y+z-1=0$ and $x+y+z+1=0$.

(A) 0 (B) 2 (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{2}{\sqrt{3}}$ (E) $\sqrt{\frac{2}{3}}$
(F) $\frac{1}{3}$ (G) $\frac{2}{3}$ (H) $\frac{\sqrt{2}}{3}$ (I) None of the above.

5. Find the distance between the two parallel lines

$$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+1}{4} \quad\text{and}\quad \frac{x+2}{1}=\frac{y+2}{2}=\frac{z}{4}$$

(A) 0 (B) 1 (C) $\sqrt{\frac{32}{7}}$ (D) $\sqrt{\frac{71}{3}}$
(E) $\frac{\sqrt{32}}{7}$ (F) $\frac{\sqrt{71}}{3}$ (G) $\frac{32}{\sqrt{7}}$ (H) $\frac{71}{3}$
(I) None of the above.

6. The curve $4z-y^2=0$ in the YZ plane is rotated about the Z-axis. What is the quadric surface generated and what is its equation?

(A) Elliptic paraboloid; $4x^2+y^2=4z$
(B) Circular paraboloid; $x^2+y^2=4z$
(C) Elliptic paraboloid; $x^2+4y^2=4z$
(D) Ellipsoid; $4x^2+4y^2+z^2=1$
(E) Hyperboloid of one sheet; $4x^2+4y^2-4z^2=1$
(F) Parabolic cylinder: $4z-y^2=0$
(G) Hyperbolic cylinder; $4z^2-y^2=0$
(H) Cone: $x^2+y^2=16z^2$
(I) None of the above.

/...3

7. For the two curves $\underset{\sim}{r}_1(t) = e^t\underset{\sim}{i} + 2\cos t\underset{\sim}{j} + (t^2-2)\underset{\sim}{k}$, $-\pi \le t \le \pi$, and $\underset{\sim}{r}_2(s) = s\underset{\sim}{i} + 2\underset{\sim}{j} + (s^2-3)\underset{\sim}{k}$, find the values of t and s at which the curves intersect and so calculate the angle between the tangents to the curves at the point of intersection.

   (A) 0 (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\cos^{-1}(\frac{1}{\sqrt{3}})$ (E) $\cos^{-1}(\frac{1}{\sqrt{5}})$
   (F) $\cos^{-1}(\frac{1}{\sqrt{6}})$ (G) π (H) $\frac{3\pi}{4}$ (I) None of the above.

8. Find the arc length of the curve

$$\underset{\sim}{r}(t) = t\underset{\sim}{i} + \underset{\sim}{j} + \left(\frac{1}{6}t^3 + \frac{1}{2}t^{-1}\right)\underset{\sim}{k}, \quad 1 \le t \le 3 .$$

   (A) $\frac{25}{6}$ (B) $\frac{14}{6}$ (C) $\frac{14}{3}$ (D) $\frac{28}{3}$ (E) 5
   (F) $\frac{29}{6}$ (G) $\frac{29}{3}$ (H) $\frac{25}{3}$ (I) None of the above.

9. Find the curvature at the vertices of the hyperbola $y^2 - x^2 = 1$.

   (A) 1 (B) ± 1 (C) 0 (D) ∞ (E) $\frac{1}{2^{3/2}}$
   (F) $\pm\frac{1}{2^{3/2}}$ (G) 2 (H) ± 2 (I) None of the above.

10. Find the spherical co-ordinates (ρ, θ, ϕ) of the point whose rectangular co-ordinates are $\left(2, 2, \frac{2\sqrt{6}}{3}\right)$

   (A) $(\frac{32}{3}, \frac{\pi}{4}, \frac{\pi}{3})$ (B) $(\sqrt{\frac{32}{3}}, \frac{\pi}{4}, \frac{\pi}{3})$ (C) $(\sqrt{\frac{32}{3}}, \frac{\pi}{4}, \frac{\pi}{6})$ (D) $(\frac{32}{3}, \frac{\pi}{4}, \frac{\pi}{6})$
   (E) $(\sqrt{\frac{32}{3}}, \frac{\pi}{3}, \frac{\pi}{4})$ (F) $(\sqrt{\frac{32}{3}}, \frac{\pi}{6}, \frac{\pi}{3})$ (G) $(\frac{32}{3}, \frac{\pi}{3}, \frac{\pi}{4})$ (H) $(\frac{32}{3}, \frac{\pi}{6}, \frac{\pi}{4})$
   (I) None of the above.

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BONUS QUESTION: 3 MARKS
Write the Answer to this question in the booklet provided.

11. Calculate the angle between the two unit vectors each of which makes an angle of $\frac{\pi}{4}$ with both of $\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{i} + \underset{\sim}{k}$.

$\theta = 180$

** THE END **

University of Saskatchewan
Department of Mathematics & Statistics
MATH 223.3 – MIDTERM TEST 2

Instructors: *(01) H. S. Yin* *(03) J. A. Brooke* *(05) P. J. Browne*
November 15, 1999 Time: 4:30–6:00 pm

- This is an open book exam. Students may use Stewart: Calculus.
- Calculators, notes, formula sheets are not permitted.
- Only a soft lead pencil should be used on the opscan sheet.
- Your student ID number should be encoded and written in the upper left hand corner of the opscan sheet.

ALL QUESTIONS ARE OF EQUAL VALUE
Section A: Answer these questions on the opscan sheet

1. Find $\displaystyle\lim_{(x,y)\to(0,0)} \left(\frac{x^3 - 2y^3 + x^2y - 2xy^2 + 3x^2 + 3y^2}{x^2+y^2}\right)$.

   (A) Does not exist (B) ∞ (C) 0
   (D) 3 (E) 2 (F) -2
   (G) 1 (H) -1 (I) None of the above.

2. Find the equations of the normal line to the surface $z = 3x^2 + 4y^2 - 6xy$ at the point on the surface where $x = 1$, $y = 1$.

   (A) $x = 0, \frac{y-2}{1} = \frac{z+1}{1}$ (B) $\frac{x}{1} = \frac{y-1}{2} = \frac{z-1}{-1}$
   (C) $\frac{x}{1} = \frac{y+1}{2} = \frac{z+1}{-1}$ (D) $\frac{x}{1} = \frac{y-2}{1} = \frac{z+1}{1}$
   (E) $x = 1, \frac{y-1}{2} = \frac{z-1}{-1}$ (F) $x = 1, \frac{y+1}{2} = \frac{z+1}{-1}$
   (G) $x = 1, \frac{y-2}{1} = \frac{z+1}{1}$ (H) $x = 0, \frac{y-1}{2} = \frac{z-1}{-1}$
   (I) None of the above.

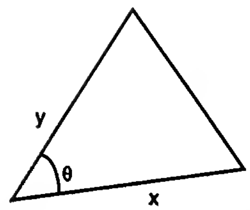
3. Use differentials to find an approximate value for $f(x,y) = x - 3y + \ln(x - 3y)$ at $(x,y) = (6.98, 2.06)$

(A) 0.60 (B) 0.62 (C) 1.40

(D) 0.68 (E) 0.32 (F) −0.4

(G) 1.32 (H) −0.38 (I) None of the above.

4. If the lengths of two sides of a triangle are x and y, and if θ is the angle between the sides, the area A of the triangle is given by $A = \frac{1}{2}xy\sin\theta$. (see figure below). Supppose the sides are each increasing at a rate of 3 cm/sec and θ is decreasing at a rate of 0.1 radians/sec. How fast is the area changing when $x = 15$ cms, $y = 20$ cms, $\theta = \frac{\pi}{4}$?



(A) $\frac{135}{2\sqrt{2}}$ (B) $-\frac{135}{2\sqrt{2}}$ (C) $-\frac{15}{2\sqrt{2}}$

(D) $\frac{75}{2\sqrt{2}}$ (E) $-\frac{45}{2\sqrt{2}}$ (F) $-\frac{75}{2\sqrt{2}}$

(G) $\frac{15}{2\sqrt{2}}$ (H) 0 (I) None of the above.

5. The curve $\underset{\sim}{r}(t) = 2t\underset{\sim}{i} + \frac{3}{t}\underset{\sim}{j} - (t^2+1)\underset{\sim}{k}$ and the surface $x^2 + y^2 + 3z^2 = 25$ intersect at the point $P(2,3,-2)$. Find the direction of the tangent to the curve at P and the direction of the normal to the surface at P, and so calculate the angle between these directions.

(A) $\cos^{-1}\left(\frac{1}{\sqrt{7}}\right)$ (B) $\cos^{-1}\left(\frac{1}{\sqrt{17}}\right)$ (C) $\cos^{-1}\left(\frac{1}{7}\right)$

(D) $\cos^{-1}\left(\frac{1}{17}\right)$ (E) $\cos^{-1}\left(\frac{1}{119}\right)$ (F) $\frac{\pi}{3}$

(G) 0 (H) $\cos^{-1}\left(\frac{7}{\sqrt{77}}\right)$ (I) None of the above.

6. Find the directional derivative of the function $f(x,y,z) = x\cos y \sin z$ at the point $\left(1, \pi, \frac{\pi}{4}\right)$ in the direction $2\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$.

(A) $-\sqrt{18}$ (B) $\sqrt{18}$ (C) $\sqrt{\frac{2}{7}}$

(D) $-\sqrt{\frac{2}{7}}$ (E) $-\sqrt{\frac{6}{7}}$ (F) $\sqrt{\frac{6}{7}}$

(G) $\sqrt{\frac{3}{7}}$ (H) $-\sqrt{\frac{3}{7}}$ (I) None of the above.

7. Find the shortest distance from the point $P(1,2,0)$ to the elliptic cone $z = \sqrt{x^2 + 2y^2}$ by minimizing the square of the distance from P to a point (x,y,z) on the cone.

(A) $\sqrt{\frac{9}{5}}$ (B) $\frac{3}{2}$ (C) $\sqrt{\frac{3}{2}}$

(D) $\sqrt{\frac{19}{5}}$ (E) $\sqrt{\frac{19}{6}}$ (F) $\frac{19}{6}$

(G) $\frac{\sqrt{19}}{6}$ (H) $\sqrt{\frac{8}{3}}$ (I) None of the above.

8. If $z = \cos(xy) + y\cos(\pi x)$, $x = u^2 + v$, $y = u - v^2$, find $\frac{\partial z}{\partial v}$ when $u = 1, v = 1$.

(A) 2π (B) -2π (C) 2

(D) π (E) 0 (F) −2

(G) $-\pi$ (H) −1 (I) None of the above.

9. Find the volume of the solid bounded above by the surface $z = x^3y$ and below by the triangle with vertices $(0,0,0)$, $(1,0,0)$ and $(0,1,0)$.

(A) $\frac{1}{5}$ (B) $\frac{1}{6}$ (C) $\frac{1}{120}$

(D) $\frac{1}{30}$ (E) $\frac{1}{60}$ (F) $\frac{5}{60}$

(G) $\frac{5}{120}$ (H) $\frac{1}{15}$ (I) None of the above.

$\iint\limits_R e^{y^2}\, dy\, dx$ where R is the triangle formed by the y-axis and the lines $y = 2x$,

$e^4 - 1)$	(B) $\frac{1}{4}\left(e^4 + 1\right)$	(C) $\frac{1}{4}e^4$
$^4 - 1)$	(E) $\frac{1}{2}\left(e^4 + 1\right)$	(F) $\frac{1}{2}e^2$
$^2 - 1)$	(H) $\frac{1}{2}\left(e^2 + 1\right)$	(I) None of the above.

Section B: Bonus Question.

he answer to the following question in the booklet provided.

$f(x, y, z) = xyz$ subject to $x^3 + y^3 + z^3 = 3$, $x \geq 0$, $y \geq 0, z \geq 0$.

** THE END **

University of Saskatchewan
Department of Mathematics & Statistics
MATH 223.3 – Final

Instructors: *(01) P. J. Browne* *(03) B. Friberg* *(05) H. Teismann*

December 9, 2000 Time: 2:00-5:00 pm

- This is an open book exam. Students may use Stewart: Calculus.
- Calculators, notes, formula sheets are not permitted.
- Only a soft lead pencil should be used on the opscan sheet.
- Your student ID number should be encoded and written, with your name, in the upper left hand corner of the opscan sheet.

ALL QUESTIONS ARE OF EQUAL VALUE

Section A: Answer these questions on the opscan sheet

1. Find the point of intersection (if it exists) of the lines

$$\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-5}{1} \quad \text{and} \quad \frac{x-3}{-1}=\frac{y}{1}, \quad z=7 .$$

(A) $(1,2,5)$ (B) $(1,2,7)$ (C) $(3,0,0)$ (D) $(-1,1,7)$
(E) $(3,0,7)$ (F) $(0,3,7)$ (G) $(0,3,0)$ (H) Lines do not intersect
(I) None of the above.

2. Find the equation of the plane passing through the point $(1,1,1)$ which is parallel to the plane containing the points $(0,1,0)$, $(1,2,-1)$ and $(-1,3,4)$.

(A) $-2x-y+z+2=0$ (B) $2x-y+z-2=0$ (C) $-2x+y+z=0$
(D) $2x-y+z+1=0$ (E) $3x-y+z-3=0$ (F) $2x+y+z-4=0$
(G) $3x+y-z-3=0$ (H) $2x+y-z-2=0$ (I) None of the above

3. Find the length of the curve given by

$$\underset{\sim}{r}(t)=<t^3,t^2,t^3>, \quad 0\le t\le 1 .$$

(A) $\frac{2}{3}[22^{3/2}-8]$ (B) $\frac{1}{36}[22^{3/2}-8]$ (C) $\frac{1}{18}[22^{3/2}-8]$
(D) $\frac{1}{18}[22^{1/2}+2]$ (E) $\frac{1}{54}[22^{3/2}+8]$ (F) $\frac{1}{18}[22^{1/2}-2]$
(G) $\frac{1}{54}[19^{3/2}-1]$ (H) $\frac{1}{54}[22^{3/2}-8]$ (I) None of the above

.../2

4. The set of all points $P(x,y,z)$ whose distance to the plane $z=3$ is twice the distance from P to $(1,1,0)$ forms an ellipsoid with equation

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1, \quad a>0, b>0, c>0 .$$

Find (x_0,y_0,z_0) and a,b,c.

(A) $(x_0,y_0,z_0)=(-1,-1,1)$, $a=3$, $b=3$, $c=4$
(B) $(x_0,y_0,z_0)=(1,1,-1)$, $a=3$, $b=3$, $c=4$
(C) $(x_0,y_0,z_0)=(-1,-1,1)$, $a=\sqrt{3}$, $b=\sqrt{3}$, $c=2$
(D) $(x_0,y_0,z_0)=(1,1,-1)$, $a=\sqrt{3}$, $b=\sqrt{3}$, $c=2$
(E) $(x_0,y_0,z_0)=(1,1,-1)$, $a=\sqrt{\frac{3}{2}}$, $b=\sqrt{\frac{3}{2}}$, $c=\sqrt{2}$
(F) $(x_0,y_0,z_0)=(1,1,-1)$, $a=\frac{3}{2}$, $b=\frac{3}{2}$, $c=2$
(G) $(x_0,y_0,z_0)=(-1,-1,1)$, $a=\sqrt{\frac{3}{2}}$, $b=\sqrt{\frac{3}{2}}$, $c=\sqrt{2}$
(H) $(x_0,y_0,z_0)=(-1,-1,1)$, $a=\frac{3}{2}$, $b=\frac{3}{2}$, $c=2$
(I) None of the above.

5. Given that the equation $z^3-xz-y=0$ implicitly defines z as a function of x and y, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, and use these to find $\frac{\partial^2 z}{\partial x \partial y}$ in terms of x,y,z.

(A) $(3z^2+x)/(3z^2-x)^2$ (B) $1/(3z^2-x)^2$ (C) $-1/(3z^2-x)^2$
(D) $-(3z^2+x)/(3z^2-x)^3$ (E) $-(3z^2+x)/(3z^2-x)$ (F) $-(3z^2+x)/(3z^2-$
(G) $-(6z^2-x)/(3z^2-x)^3$ (H) $(6z^2+x)/(3z^2+x)^3$ (I) None of the above

6. A particle starts at time $t=0$ from position $(1,1)$ with speed 2m/sec in the negative Y-direction. It is subject to an acceleration given by

$$\underset{\sim}{a}(t)=\frac{1}{\sqrt{t+1}}\underset{\sim}{i}+6t\underset{\sim}{j} .$$

Find its position at $t=3$.

(A) $(13/3,22)$ (B) $(5/3,21)$ (C) $(5,22)$ (D) $(32/3,27)$
(E) $(17/3,20)$ (F) $(7/3,27)$ (G) $(7/3,22)$ (H) $(16/3,27)$
(I) None of the above

.../3

7. Find the equation of the tangent plane to the surface $xyz^3 + yz^2 = 4$ at the point $(1,2,1)$.

(A) $x+y+5z-8=0$ (B) $2x+2y+5z-11=0$
(C) $x+y+4z-7=0$ (D) $x+y+2z-5=0$
(E) $2x+y+10z-14=0$ (F) $x+2y+10z-15=0$
(G) $x-y+5z-6=0$ (H) $x+y+10z-13=0$
(I) None of the above

8. Find the directional derivative of $f(x,y) = \arctan(xy)$ at the point $(1,2)$ in the direction of the line $y = 2x$ for increasing x.

(A) $\frac{-4}{5}$ (B) $\frac{-4}{5\sqrt{5}}$ (C) $\frac{-1}{\sqrt{5}}$ (D) 0 (E) $\frac{2}{5\sqrt{5}}$
(F) $\frac{4}{5\sqrt{5}}$ (G) $\frac{4}{5}$ (H) $\frac{1}{\sqrt{5}}$ (I) None of the above

9. Evaluate $\iint_R (x+y)dA$ where R is the region bounded by $y=0$ and $y=\sqrt{4-x^2}$.

(A) 4/3 (B) −16 (C) −16/3 (D) 0 (E) 16/3
(F) 8 (G) 16 (H) 8/3 (I) None of the above

10. Find the maximum value of $f(x,y,z) = 2x+2y+4z$ subject to $x^2+y^2+z^2=6$.

(A) $\frac{2\sqrt{6}}{3}$ (B) 6 (C) 4 (D) 12 (E) −12
(F) 2 (G) $4\sqrt{6}$ (H) −2 (I) None of the above

11. A lamina occupies the region inside the circle $x^2+y^2=1$ in the first quadrant. Its density at any point is proportional to the square of the distance from $(0,0)$. Find the co-ordinates of its centre of mass.

(A) $\left(\frac{8}{5\pi}, \frac{8}{5\pi}\right)$ (B) $\left(\frac{\pi}{40}, \frac{\pi}{40}\right)$ (C) $\left(\frac{5\pi}{8}, \frac{5\pi}{8}\right)$
(D) $\left(\frac{40}{\pi}, \frac{40}{\pi}\right)$ (E) $\left(\frac{4}{5\pi}, \frac{4}{5\pi}\right)$ (F) $\left(\frac{5}{4\pi}, \frac{5}{4\pi}\right)$
(G) $\left(\frac{2}{5\pi}, \frac{2}{5\pi}\right)$ (H) $\left(\frac{5}{2\pi}, \frac{5}{2\pi}\right)$ (I) None of the above

12. Evaluate $\int_0^1 \int_0^x \frac{1}{\sqrt{1-y^2}}\,dy\,dx$

(A) 2 (B) 1 (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{2}-1$ (E) $\frac{\pi}{2}+1$
(F) $\pi+1$ (G) $\pi-1$ (H) $\pi+2$ (I) None of the above

13. Find the surface area of the surface $z = \sqrt{3}x + \sqrt{3}\,y^2$ lying above the triangle with vertices at $(0,0,0)$, $(1,1,0)$ and $(0,1,0)$.

(A) 2/9 (B) 2 (C) 7/3 (D) 16/9 (E) 14/9
(F) 7/9 (G) 16/3 (H) 14/3 (I) None of the above

14. Calculate $\iiint_R z\,dV$ where R is the region in the first octant bounded by $y^2+z^2=1$, $y=x$, $z=0$, $x=0$.
[Hint: set up the integral in the form $\iiint z\,dz\,dx\,dy$ with appropriate terminals for the integrations.]

(A) $\frac{7}{4}$ (B) $\frac{7}{3}$ (C) $\frac{7}{12}$ (D) $\frac{7}{6}$ (E) $\frac{3}{8}$
(F) $\frac{3}{4}$ (G) $\frac{1}{4}$ (H) $\frac{1}{8}$ (I) None of the above

15. Calculate $\iint_R \frac{x}{x^2+y^2}\,dA$ where R is the region outside $x^2+y^2=1$ and inside $x^2+y^2=2y$.

(A) 3/8 (B) 1/4 (C) 3/2 (D) 0 (E) 1/8
(F) 1 (G) 3/4 (H) 1/2 (I) None of the above

16. Use spherical co-ordinates to calculate $\iiint_R (x^2+y^2+z^2)dV$ where R is the region whose lower boundary is the cone $z=\sqrt{x^2+y^2}$ and whose upper boundary is $x^2+y^2+z^2=1$.

(A) $\frac{2\pi}{5}\left(1-\frac{1}{\sqrt{2}}\right)$ (B) $\frac{2\pi}{5}$ (C) $\frac{\pi}{5}$ (D) $\frac{2\pi}{3}\left(1-\frac{1}{\sqrt{2}}\right)$ (E) $\frac{2\pi}{3}$
(F) $\frac{\pi}{3}$ (G) $\frac{\pi}{2}\left(1-\frac{1}{\sqrt{2}}\right)$ (H) $\frac{\pi}{2}$ (I) None of the above

17. Find the work done by the force

$$\underset{\sim}{F}(x,y,z) = \frac{1}{x^2+(y+1)^2+(z+2)^2} < 1,2,3 >$$

acting on a particle which moves in a straight line from $(1,1,1)$ to $(2,3,4)$.

(A) 1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) -1 (E) 0

(F) $\frac{\sqrt{14}}{2}$ (G) $\sqrt{14}$ (H) $\frac{1}{2\sqrt{14}}$ (I) None of the above

18. Find the line integral $\int_C x\,ds$ where C is the curve $y = x^2,\ z = x^2$ from $(0,0,0)$ to $(1,1,1)$.

(A) 13/12 (B) 29/24 (C) 39/24 (D) 13/24 (E) 9/8

(F) 13/3 (G) 13/6 (H) 29/6 (I) None of the above

19. Find the integral $\int_C \arctan y\,dx - \frac{y^2x}{1+y^2}\,dy$ where C is the boundary of the square with vertices $(0,0), (0,1), (1,1), (1,0)$ described in an counterclockwise fashion.

(A) 1 (B) 0 (C) 2π (D) $\pi/4$ (E) -1

(F) $-\pi$ (G) π (H) $-\pi/4$ (I) None of the above

20. Calculate $\int_C \underset{\sim}{F}\cdot d\underset{\sim}{r}$ where $\underset{\sim}{F}(x,y) =< e^y + ye^x,\ xe^y + e^x >$ and C is the curve $\underset{\sim}{r}(t) =< \sin\left(\frac{\pi t}{2}\right),\ \ln t >,\ 1 \le t \le 2$.
[Use the Fundamental Theorem for Line Integrals]

(A) $2\ln 2 - 1$ (B) $2\ln 2 + 2$ (C) $2\ln 2 - 2$ (D) 1 (E) 2

(F) $\ln 2 + 1$ (G) $\ln 2$ (H) $\ln 2 - 1$ (I) None of the above

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Section B: Bonus Questions.

Write complete solutions to these problems in the answer booklet provided.

B1. A solid has lower boundary given by the cone $z = \sqrt{x^2+y^2}$ and upper boundary the plane $z = 3$. Its density is given by $f(x,y,z) = 3 - z$. Find its mass.
[Use cylindrical co-ordinates]

B2. Find the point on the plane $4x + 3y + z = 12$ that is closest to $(2,-3,0)$.

** THE END **

University of Saskatchewan
Department of Mathematics & Statistics
MATH 223.3 – Final

Instructors: *(01) H. S. Yin* *(03) J. A. Brooke* *(05) P. J. Browne*

December 9, 1999 Time: 2:00-5:00 pm

- This is an open book exam. Students may use Stewart: Calculus.
- Calculators, notes, formula sheets are not permitted.
- Only a soft lead pencil should be used on the opscan sheet.
- Your student ID number should be encoded and written, with your name, in the upper left hand corner of the opscan sheet.

ALL QUESTIONS ARE OF EQUAL VALUE

Section A: Answer these questions on the opscan sheet

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1. Find the equation of the plane containing the points $(2,0,1)$ and $(3,1,2)$ and parallel to the line through $(6,7,0)$ and $(4,7,1)$.

(A) $x+3y+2z-4=0$ (B) $x-3y+2z-11=0$ (C) $x-3y+2z+11=0$
(D) $x-3y+2z-15=0$ (E) $x-3y+2z+15=0$ (F) $x-3y-2z=0$
(G) $x-3y+2z+4=0$ (H) $x-3y+2z-4=0$ (I) None of the above

2. Find the equations of the line through $(1,1,1)$ and parallel to the intersection of $2x+3y+z=5$ and $x+y-z=1$.

(A) $\frac{x-1}{2}=\frac{y-1}{3}, z=0$ (B) $\frac{x-1}{4}=\frac{y-1}{-3}, z=1$ (C) $\frac{x+1}{4}=\frac{y+1}{-3}, z=$
(D) $\frac{x-1}{-2}=\frac{y-1}{3}, z=0$ (E) $\frac{x-1}{4}=\frac{y-1}{-3}=\frac{z-1}{1}$ (F) $\frac{x+1}{-4}=\frac{y+1}{3}=\frac{z-}{}$
(G) $\frac{x+1}{-2}=\frac{y+1}{3}, z=0$ (H) $\frac{x+1}{4}=\frac{y+1}{-3}=\frac{z+1}{1}$ (I) None of the above

3. Find an equation for the surface obtained by rotating the parabola $x=y^2$ about the x-axis.

(A) $x=y^2+z^2$ (B) $x^2=y^2+z^2$ (C) $y^2+z^2-x^2=1$
(D) $x^2=y^2-z^2$ (E) $x=y^2-z^2$ (F) $x^2-y^2-z^2=1$
(G) $x^2=z^2-y^2$ (H) $x=z^2-y^2$ (I) None of the above

4. Calculate the length of the curve given by

$$\underset{\sim}{r}(t) = < e^t \cos t, e^t \sin t, e^t >, \ 0 \le t \le \ln 5 .$$

(A) $\sqrt{3}$	(B) 36	(C) 15
(D) $4\sqrt{3}$	(E) $5\sqrt{3}$	(F) 12
(G) 4	(H) 5	(I) None of the above

5. A particle moves so that $\underset{\sim}{r}(t) = 2\underset{\sim}{i} + t^2\underset{\sim}{j} + (t-1)^2\underset{\sim}{k}$. At what time is its speed a minimum and what is the minimum speed?

(A) $t = 1$, speed $= 2$	(B) $t = 1$, speed $= 4$	(C) $t = \frac{1}{2}$, speed $= \sqrt{2}$
(D) $t = 0$, speed $= 2$	(E) $t = 0$, speed $= 4$	(F) $t = \frac{1}{2}$, speed $= 2$
(G) $t = 2$, speed $= 2\sqrt{7}$	(H) $t = \frac{1}{2}$, speed $= 1$	(I) None of the above

6. Find the directional derivative of $f(x,y,z) = x^2 + 2xyz - yz^2$ at $(1,1,2)$ in a direction parallel to the straight line $\frac{x-1}{4} = \frac{z}{-3}$, $y = 1$.

(A) $\frac{6}{5}$	(B) $\frac{31}{\sqrt{40}}$	(C) $\frac{31}{\sqrt{26}}$
(D) 30	(E) $\frac{30}{7}$	(F) 5
(G) $\frac{15}{\sqrt{10}}$	(H) 6	(I) None of the above

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7. If z is defined as a function of x and y by means of the equation $xe^y + ye^z + ze^x = 1$, find $\frac{\partial z}{\partial y}$ in terms of x, y and z.

(A) $\frac{e^y + ze^x}{e^x + ye^z}$	(B) $\frac{-ze^x}{e^x + ye^z}$	(C) $\frac{-e^y}{e^x + ye^z}$
(D) $\frac{-e^y - ze^x}{e^x + ye^z}$	(E) $\frac{-e^z - xe^y}{e^x + ye^z}$	(F) $\frac{-xe^y}{e^x + ye^z}$
(G) $\frac{e^z + xe^y}{e^x + ye^z}$	(H) $\frac{-e^z - xe^y}{e^x}$	(I) None of the above

.../3

8. Find the point(s) on the surface $xy + \frac{1}{x} + \frac{1}{y} - z = 0$ where the tangent plane is horizontal.

(A) Tangent plane is never horizontal	(B) $(1,-1,-1), (-1,1,-1)$
(C) $(1,1,3), (0,0,0)$	(D) $(1,1,1), (-1,-1,-1)$
(E) $(1,1,1)$	(F) $(1,1,3)$
(G) $(1,1,3), (-1,-1,-1)$	(H) $(1,1,0)$
(I) None of the above	

9. Find the maximum value of $f(x,y,z) = xyz^2$ subject to $x + y + z = 20$, $x \ge 0$, $y \ge 0$, $z \ge 0$.

(A) 1024	(B) 10,000	(C) No maximum exists
(D) 1152	(E) $\frac{8000}{81}$	(F) 625
(G) 250	(H) 2500	(I) None of the above

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10. Find and classify the stationary (critical) points of $f(x,y) = x^3 + y^3 - 3xy$.

(A) $(0,0)$ saddle; $(1,1)$ local min
(B) $(0,0)$ local max; $(1,1)$ local min
(C) $(0,1)$ saddle; $(1,0)$ saddle; $(0,0)$ saddle; $(1,1)$ local min
(D) $(0,0)$ saddle; $(1,1)$ local max
(E) $(0,1)$ local max; $(1,0)$ local min
(F) $(0,1)$ local min; $(1,0)$ local max
(G) $(0,0)$ local max; $(1,1)$ saddle
(H) $(0,0)$ saddle
(I) None of the above

11. By interchanging the order of integration evaluate $\int_0^1 \int_x^1 \cos\left(\frac{\pi y^2}{2}\right) dy\, dx$.

(A) $-\frac{1}{\pi}$	(B) -1	(C) $\frac{1}{2\pi}$
(D) $\frac{1}{\pi}$	(E) 1	(F) 0
(G) $\frac{2}{\pi}$	(H) $\frac{1}{2}$	(I) None of the above

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.../4

12. Find the volume of the solid bounded by the circular cylinder $x^2+y^2=1$, the plane $z=0$ and the plane $x+z=1$.

(A) $\frac{\pi}{3}$ (B) 2π (C) π
(D) $\frac{1}{6}$ (E) 2 (F) 1
(G) $\frac{1}{2}$ (H) $\frac{\pi}{2}$ (I) None of the above

13. Calculate $\iiint\limits_D x^2\,dV$ where D is the tetrahedron in the first octant bounded by the co-ordinate planes and the plane $x+y+z=1$.

(A) $\frac{1}{60}$ (B) $\frac{1}{30}$ (C) $\frac{1}{120}$
(D) $\frac{1}{15}$ (E) $\frac{2}{15}$ (F) $\frac{1}{5}$
(G) 1 (H) $\frac{1}{3}$ (I) None of the above

14. Calculate the area of that part of the surface $z=6-x^2-y^2$ lying over the annulus $2\le x^2+y^2\le 6$.

(A) $\frac{71\pi}{3}$ (B) $\frac{71\pi}{6}$ (C) $\frac{49\pi}{24}$
(D) $\frac{49\pi}{12}$ (E) $\frac{98\pi}{3}$ (F) $\frac{49\pi}{3}$
(G) $\frac{49\pi}{48}$ (H) $\frac{49\pi}{6}$ (I) None of the above

15. Calculate $\iiint\limits_D e^{(x^2+y^2+z^2)^{3/2}}\,dx\,dy\,dz$ where D is the solid whose lower boundary is the surface $z=\sqrt{x^2+y^2}$ and whose upper boundary is the surface $x^2+y^2+z^2=1$.

(A) $\frac{\pi^3}{6\sqrt{2}}$ (B) $\frac{\pi^2}{48}(e-1)\frac{1}{\sqrt{2}}$ (C) $\frac{\pi^2}{24}(e-1)\frac{1}{\sqrt{2}}$
(D) $\frac{\pi^2}{6}(e-1)$ (E) $\frac{2\pi}{3}(e-1)\left(1-\frac{1}{\sqrt{2}}\right)$ (F) $\frac{\pi^2 e}{6}\left(1-\frac{1}{\sqrt{2}}\right)$
(G) $\frac{\pi^2}{12}e\frac{1}{\sqrt{2}}$ (H) $\frac{\pi^2}{12}(e-1)\left(1-\frac{1}{\sqrt{2}}\right)$ (I) None of the above

.../5

16. Calculate $\iint\limits_D xy\,dx\,dy$ where D is the region in the first quadrant bounded by the curves $x^2+y^2=4$, $x^2+y^2=9$, $x^2-y^2=1$, $x^2-y^2=4$. [Use the change of variables: $u=x^2+y^2$, $v=x^2-y^2$]

(A) $-\frac{15}{8}$ (B) $\frac{15}{8}$ (C) $\frac{3}{8}$
(D) $-\frac{3}{8}$ (E) $\frac{15}{16}$ (F) $\frac{3}{16}$
(G) $-\frac{15}{16}$ (H) $-\frac{3}{16}$ (I) None of the above

17. Calculate the line integral $\int\limits_C \underset{\sim}{F}\cdot d\underset{\sim}{r}$ where $\underset{\sim}{F}(x,y,z)=xy\underset{\sim}{i}+yz\underset{\sim}{j}+xz\underset{\sim}{k}$ and C is the path given by $\underset{\sim}{r}(t)=t\underset{\sim}{i}+t^2\underset{\sim}{j}+t^3\underset{\sim}{k}$ from $(-1,1,-1)$ to $(1,1,1)$.

(A) $\frac{10}{7}$ (B) $\frac{5}{7}$ (C) $\frac{27}{14}$
(D) $\frac{13}{14}$ (E) $\frac{6}{7}$ (F) $\frac{2}{7}$
(G) 1 (H) $\frac{12}{7}$ (I) None of the above

18. Calculate the work done by a force $\underset{\sim}{F}(x,y,z)=x\underset{\sim}{i}+xy\underset{\sim}{j}+xyz\underset{\sim}{k}$ applied to an object that moves in a straight line from $(0,1,4)$ to $(1,0,-4)$.

(A) $\frac{193}{3}$ (B) $-\frac{193}{3}$ (C) $\frac{2}{3}$
(D) $\frac{1}{3}$ (E) $-\frac{1}{3}$ (F) $\frac{97}{3}$
(G) $-\frac{97}{3}$ (H) 0 (I) None of the above

.../5

19. A force $\underset{\sim}{F} = -\frac{1}{2}(y\underset{\sim}{i} - x\underset{\sim}{j})$ is applied to a particle which moves in an elliptical orbit $\underset{\sim}{r}(t) =< a\cos t, b\sin t >$; a, b constants. Find the work done as the particle makes one complete orbit.

✓

(A) 0	(B) a^2b^2	(C) $\frac{a^2b^2}{2}$
(D) ab	(E) $\frac{\pi ab}{2}$	(F) πa^2b^2
(G) πab	(H) $\frac{\pi a^2b^2}{2}$	(I) None of the above

20. The solid sphere whose boundary has equation in spherical coordinates $\rho = 2\cos\phi$, has density given by $f(\rho, \theta, \phi) = \rho$. Calculate its mass.

X

(A) $\frac{4}{5}$	(B) $\frac{2}{5}$	(C) 4π
(D) $\frac{\pi}{5}$	(E) $\frac{8\pi}{5}$	(F) 8π
(G) $\frac{2\pi}{5}$	(H) 2π	(I) None of the above

Section B: Bonus Questions.

Write the answers to the following questions in the booklet provided.

B1. Find the distance from the point $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ to the sphere $x^2 + y^2 + z^2 = 1$.

B2. Calculate the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ by setting $x = a\rho\sin\phi\cos\theta$, $y = b\rho\sin\phi\sin\theta$, $z = c\rho\cos\phi$.

** THE END **

UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE271.3

Midterm Examination
Part B

Instructor: S.O. Kasap — November 6, 2002
Time allowed: Part B is nominally 1 hour.
Total time allowed: 2 hours for Parts A and B.

Instructions: Open book examination. *Answer any 2 questions from 3 questions.* All questions carry equal marks. Marks for part–questions are shown in [ ] in the left hand margin. All answers must be given in conventional units. State clearly all assumptions made in your derivations. All answers must be given in conventional units. Method of solution must be clearly shown. Numerical mistakes, incorrect, unconventional or missing units will be heavily penalized. Mention the source of materials data used. Unless otherwise stated all % are in weight % ($^w/_o$).

Important: You must hand in Part A before you can start Part B. Write your answers in the university answer book.

Note: You may spend more or less time on Part B; but the total exam time is 2 hours.

---

[9] 1. (a) Nickel (Ni) is a metal with the FCC crystal structure. The radius of the Ni atom is 0.1246 nm. The atomic mass of the Ni atom is 58.69 amu (g mol^{-1}). Calculate the number of Ni atoms per unit volume and density of Ni. Calculate the atomic packing factor of the Ni crystal as %.

[6] (b) Calculate the specific heat capacity of the Ni crystal.

[10] (c) The energy required to create a vacancy in the Ni crystal is 1.4 eV. The thermal expansion coefficient is about 14×10^{-6} K^{-1}. Calculate the concentration of vacancies in the Ni crystal at 700 °C.

---

[12] **2.** (a) The elastic modulus and Poisson's ratio of steel are 210 GPa and 0.27. Consider a 10 meter steel wire of diameter 2.5 mm that is carrying a tensile load of 800 N (equivalent to a mass of about 80 kg, a typical mass of an adult male). The yield strength and the tensile strength of this particular steel are 220 MPa and 400 MPa respectively . What is the new length and diameter of the steel wire? Compare the true and engineering strains and also true and engineering stresses. What is the mass required to fracture the wire?

[13] (b) A tension test is carried out on an steel specimen which has an original diameter of 12.80 mm. After fracture, the diameter at the fractured neck is measured to be 10.4 mm. The original length between two points on the specimen (between gaugē marks) is 5.00 cm. After the fracture, this length is 6.10 cm. The maximum load during the test was 55 kN

whereas at fracture the load was 44 kN.

(i) What is the tensile strength?
(ii) What is the true stress at fracture?
(iii) What is the true strain at fracture?
(iv) What is the ductility of the sample?

---

[25] 3. (a) A seemingly new radioactive metallic element, called Pz has been discovered on Planet Z^3 by space travelers. The element is believed to have a cubic structure. The atomic mass of the element has been measured to be 209 amu X-diffraction experiments carried out on this new crystal using an X-ray beam with a wavelength $\lambda = 0.1542$ nm (CuKα emission) have given the first six diffraction angles listed in the table below. What is the crystal structure and the lattice parameter? What is the density of the Pz ?

Observed diffraction peaks from an unknown metal cubic crystal.

Peak (first, second etc.)	1	2	3	4	5	6
Diffraction angle, 2θ	26.5°	37.9°	46.8°	54.6°	61.7°	68.4°

SOME CONVERSION FACTORS

LENGTH
1m = 39.37in = 3.280ft = 6.2137×10^{-4} miles
1 in = 0.0254m

ENERGY
1 kJ mole^{-1} = 0.2389 kcal mol^{-1} = 0.010363 eV atom^{-1}
1 kcal mole^{-1} = 4.1840 kJ mol^{-1} = 0.043360 eV atom^{-1}
1 eV atom^{-1} = 96.490 kJ mol^{-1} = 23.062 kcal mol^{-1}
1 ft lb = 1.356 J — 1 BTU = 1055 J
1 erg = 10^{-7} J — 1 kWh = 3.600×10^{6} J

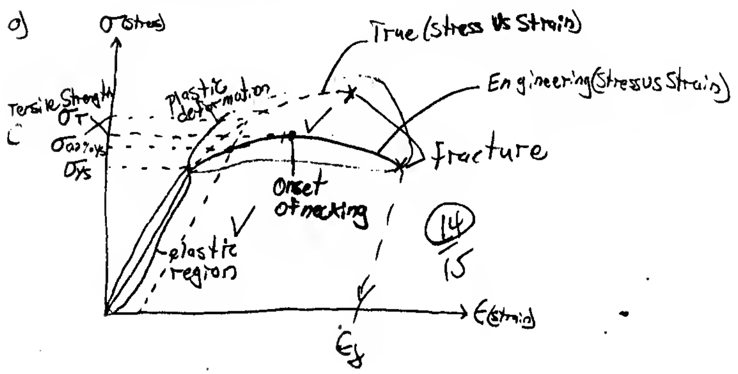
FORCE
1 N =0.2248 lb — 1 lb = 4.448 N

PRESSURE
1 Pa =1 N.m^{-2} =1.45×10^{-4} psi = 9.869×10^{-6} atm.
1 atm. = 1.013×10^{5} Pa = 1.01325 bar = 760 torr (mm Hg)
1 psi = 6.895×10^{3} Pa

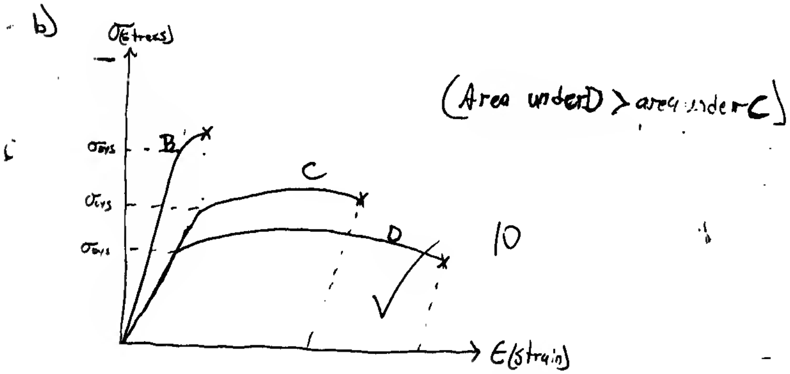
Pr

1. (a) Sketch schematically the engineering stress vs. engineering strain, and true stress vs. true strain behavior of an aluminum alloy specimen under a tension test (use the same σ–ε axes for both). Assume the specimen is eventually fractured. Identify on your sketches, the elastic region, 0.2% offset yield strength, plastic deformation region, tensile strength, the onset of necking, and fracture.
(b) Sketch schematically, <u>all on the same graph</u>, the stress vs. strain behavior of three different metals labeled as B, C and D that have the following properties.
B is brittle but C and D are ductile.
B is the strongest (highest strength).
D is more ductile than C
C is stronger than D
D is tougher than C.

a)



b)



UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE271.3

<u>Midterm Examination</u>
<u>Part A</u>

Instructor: S.O. Kasap — November 6, 2002
Time allowed: Part A is nominally 1 hour.
Total time allowed: 2 hours for Parts A and B.

Instructions: Closed book examination. *Answer any 2 questions from 4 questions.* If you answer more than 2 questions, only the first three will be marked. All questions carry equal marks. Marks for part-questions depend on the relative difficulty. All answers must be given in conventional units. All sketches must be clearly labeled and self-explanatory. Diagrams that are not properly and clearly labeled and are subject to ambiguity will be heavily penalized. Next to each diagram write short explanations that provide the key concepts and principles on which the diagram is based. Wrong concepts will be heavily penalized. State clearly all assumptions made in your derivations.

Important: You must hand in Part A before you can start Part B.

Note: You may spend more or less time on Part A; but the total exam time is 2 hours.

Question	Marks
1	24
2	23
3	
4	
5	
TOTAL	47/50

Excellant work
Well done
SOK

3. Sketch schematically the following in a crystal:
   (a) A vacancy. What is the significance of vacancies?
   (b) A large substitutional impurity and a small substitutional impurity.
   (c) An interstitial impurity.
   (d) Sketch schematically an edge dislocation in a crystal.
   (e) Sketch schematically Schottky and Frenkel defects in an ionic crystal (e.g. NaCl).
   (f) Two possible (but distinctly different) ionized substitutional impurity defects in an ionic crystal.
   Sketch schematically the structure of
   (g) a disordered substitutional solid solution,
   (h) an ordered substitutional solid solution,
   (i) an interstitial solid solution

2. (a) Figure Q.2 (a) to (d) show atomic planes in a cubic crystal. For each of the planes find the corresponding millar indices.

Miller Indizes

$\frac{1}{x}\ \frac{1}{y}\ \frac{1}{z}$

(1 0 0) (1 1 0) (1 1 1) (0 0 1) ✓

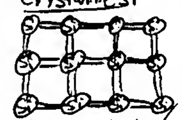
4

(b) Sketch schematically in two-dimensions the structure of
   (i) Crystalline Si
   (ii) a-Si
   (iii) a-Si:H

(c) Explain briefly with a clear diagram how a-Si:H is fabricated and its properties and uses.

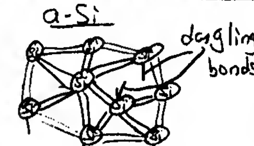
EE uses = for a bond

b) Crystalline Si



(Long Range order) ✓

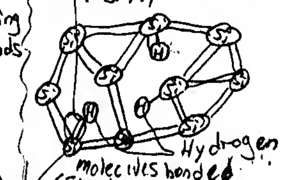
a-Si



(Short Range order) ✓

10/12

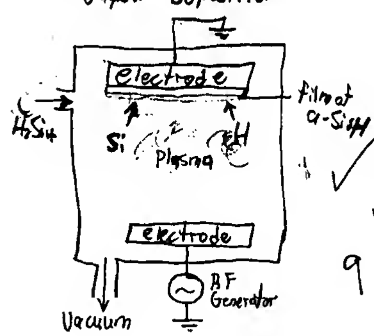
a-Si:H



(Short Range Order)

c)

Plasma Enhanced Chemical Vapour Deposition



Hydrogenated amorphous Silica is produced using chemical vapour deposition. It exhibits short range order and is use d in things like ~~solar~~ cells. a-Si:H is easier to produce in large areas compared to Si or a-Si. -

wwed

9

UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE271.3

Final Examination
Part B

Instructor: S.O. Kasap
Time allowed: Part B is nominally $1^{1}/_{2}$ hour.
Total time allowed: 3 hours for Parts A *and* B.

December 18, 2002

Instructions: Open book examination. The course textbook *and* one three-ring type binder of any size containing student-selection of notes and course material are allowed. Calculators are also allowed.

Answer any 3 questions from 5 questions. If you answer more than 3 questions, only the first three will be marked. All questions carry equal marks. All answers must be given in conventional units. State clearly all assumptions made in your derivations. Method of solution must be clearly shown. Numerical mistakes, incorrect, unconventional or missing units will be heavily penalized. Mention the source of materials data used.

Important: You must hand in Part A before you can start Part B. Write your answers in the university answer book.

Note: You may spend more or less time on Part B; but the total exam time is 3 hours.

---

[3] **1. a** KCl has the same crystal structure as NaCl. The lattice parameter a of KCl is 0.629 nm. The atomic masses of K and Cl are 39.10 g mol^{-1} and 35.45 g mol^{-1} respectively. Calculate the density of KCl.

[6] **b** Calculate the specific heat capacity of a 62 wt.% Sn and 38 wt. %Pb solder given the atomic masses118.7 and 207.2 g mol^{-1} for Sn and Pb respectively.

[8] **c** A cold worked Al-alloy component takes 200 minutes to recrytallize at 250 °C but takes 14 minutes at 276 °C. How long will the recrystallization take place at 320 °C. Can you cold work or strain harden this component at 320 °C? Why can't you cold work Pb and Sn at room temperature?

---

[7] **2. a** Consider a copper wire with a diameter of 5 mm that is carrying a current of 4 A at room temperature.

(i) Calculate the drift velocity of the conduction electrons. How does this compare with the mean velocity of the conduction electrons?

(ii) Calculate the mean free time between collisions (*i.e.* average time it takes for an electron to travel from one collision to another collision with a vibrating metal ion). What would happen to this mean free time if we were to add impurities into this metal?

(iii) Calculate the Joule heating of the wire per unit volume as W per mm^3 at 20 °C.

(iii) Calculate the Joule heating per unit volume at 50 °C.

? [10] **b** A bronze component has the composition 94wt.%Cu-6wt.%Sn. Calculate its resistivity (ρ), and thermal conductivity (κ) and estimate its TCR (α) all at 20 °C.

---

4 Sketch schematically the following

(a) The number of atoms per unit energy vs. energy for gas atoms (e.g. neon gas) in a cylinder at two temperatures T_1 and $T_2 > T_1$. What is the average energy? Identify how you would find the number of atoms with energies greater than E_A? (assume E_A >> average energy).

(b) Consider the x-ray diffraction studies of crystal structures. Explain with clear diagrams, including the diffraction pattern,

(i) the Laue technique, and
(ii) the powdered crystal technique.

(c) Sketch schematically, in two dimensions, the surface of a Si crystal that has not been passivated to show the types of surface defects that exist on the surface of a crystal.

[17] A device engineer fabricates a metal-semiconductor diode. The metal is a thin film of Al (not necessarily pure Al) on a Si crystal wafer. Suppose that the Al film is 0.15 μm thick and the Si crystal wafer is 300 μm thick. ~~Estimate the temperature induced total strain in the Al film if the device is taken~~ from ~~25 °C down to −60~~ °C. ~~What is the stress if the deformation were elastic? The yield strength~~ of this Al-~~alloy is 50 MPa~~. ~~What is the mechanical strain that results in yield (plastic deformation)? What is the mechanical strain in the Al film~~? ~~What is the stress in the Al film~~? Do you think the deformation of the Al thin film is elastic? Estimate the temperature at which the Al film begins to deform plastically (reaches the yield point)?

Over the temperature range of interest aluminum has a thermal expansion coefficient of about 10×10^{-6} K^{-1} and silicon has a thermal expansion coefficient of about 1×10^{-6} K^{-1}. The elastic modulus of Al is 70 GPa.

A power transistor with maximum power rating of 25 W has a maximum junction temperature of 140 °C. (Maximum rated power refers to the transistor case at 25 °C.)

[4] a What is the required thermal resistance between the case and the ambient for operation at maximum rated power?

[3] b What is the required thermal resistance from case to ambient if the transistor is to be operated at 20 W?

[10] c The transistor is mounted on a heat sink using a mica washer (an electrical insulator) and thermal paste (grease). The heat sink has a thermal resistance of 2.3 °C/W. The thermal resistance of the mica and the thermal paste together is 0.4 °C/W. The power amplifier circuit using this transistor is to be placed in a box in which the maximum ambient temperature is expected to reach 60 °C. What is the maximum power that can be dissipated by the transistor?

PHYSICAL CONSTANTS AND USEFUL INFORMATION

$c = 2.9979 \times 10^{8}$ m s^{-1} | $N_A = 6.0221 \times 10^{23}$ mol^{-1}

$e = 1.6021 \times 10^{-19}$ C | $k = 1.3807 \times 10^{-23}$ J K^{-1}

$m_e = 9.1091 \times 10^{-31}$ kg | $\varepsilon_o = 8.8542 \times 10^{-12}$ F m^{-1}

$h = 6.62608 \times 10^{-34}$ J s | $\mu_o = 4\pi \times 10^{-7}$ H m^{-1}

$\hbar = h/(2\pi) = 1.05459 \times 10^{-31}$ J s

Gas constant, $R = N_A k = 8.3144$ J K^{-1} mol^{-1} = 0.083144 L.bar K^{-1} mol^{-1}

Mass of proton = 1.67495×10^{-27} kg

Mass of hydrogen atom = 1.6736×10^{-27} kg

Acceleration due to gravity (at 45° latitude), g = 9.81 m s^{-2}

3. Consider a spherical pressure vessel that is to be used in aerospace applications; for example, storing high pressure hydrogen gas for a fuel cell. The stress σ in the wall is related to the pressure p in the vessel by

$$\sigma = \frac{pr}{2t} \qquad (3.1)$$

where r is the outer radius and t is the wall thickness as shown in Figure Q.3.. The mass M of the vessel is

$$M = 4\pi r^2 t\rho \qquad (3.2)$$

where ρ is the density. The stress σ in the wall will always have to satisfy

$$\sigma \le \frac{\sigma_{YS}}{S} \qquad (3.3)$$

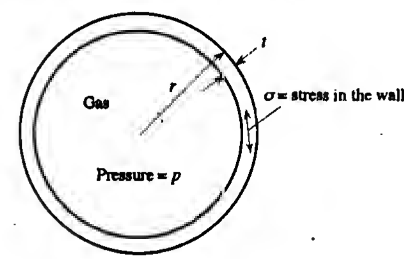
where S is a design safety factor.

[5] a Show that the *minimum mass* of the pressure vessel will be

$$M_{min} = 2S\pi p r^3 \left(\frac{\rho}{\sigma_{YS}}\right) \qquad (3.4)$$

[6] b Table Q.3 shows various materials as candidates for the vessel. By making an appropriate table, decide which material will produce the lightest vessel?

[6] c Make a relative cost table for the vessel based on the materials in Table Q.3 and hence select the material for the minimum cost vessel.



Pressure vessel (schematic sketch)

Figure Q.3

TABLE Q.3: Properties of four candidate materials for a pressure vessel
[Note: 1 tonne = 1000kg; 1g.cm^{-3} = 1 Mg m^{-3} = 10^3 kg.m^{-3}; 1 UK£ ≈ 2.5 Cnd$]

MATERIAL	Density	Elastic Modulus, E	Yield Strength, σ_{YS}	Price (Yesterday's)
	(g.cm^{-3})	(GPa)	(MPa)	(UK£/tonne)
Mild steel	7.8	207	220	200
High strength steel alloy	7.8	207	400	950
Aluminum alloy	2.7	69	193	1,000
GFRP (Glass fiber	1.8	15	75	1,300

UNITS

SI UNITS

Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K

DERIVED SI UNITS

Electric charge	coulomb	$C = A\ s$
Electrical resistance	ohm	$\Omega = V/A = kg\ m^2\ A^{-2}\ s^{-3}$
Electrical conductance	siemen	$S = 1/\Omega$
Electrical capacitance	farad	$F = A\ s\ V^{-1} = A^2\ s^4\ kg^{-1}\ m^{-2}$
Electrical inductance	henry	$H = V\ s\ A^{-1} = kg\ m^2\ s^{-1}\ A^{-2}$
Energy	joule	$J = kg\ m^2\ s^{-2} = N\ m$
Force	newton	$N = kg\ m\ s^{-2}$
Magnetic flux	weber	$Wb = V\ s = kg\ m^2\ A^{-1}\ s^{-2}$
Magnetic flux density	tesla	$T = Wb\ m^{-2} = V\ s\ m^{-2} = kg\ A^{-1}\ s^{-2}$
Pressure	pascal	$Pa = N\ m^{-2}$
Power	watt	$W = J\ s^{-1} = kg\ m^2\ s^{-3}$
Electric potential difference	volt	$V = N\ m\ C^{-1} = kg\ m^2\ s^{-3}\ A^{-1}$
Frequency	hertz	$Hz = s^{-1}$

SOME CONVERSION FACTORS

LENGTH

$1m = 39.37in = 3.280ft = 6.2137 \times 10^{-4}$ miles

$1\ in = 0.0254m$

ENERGY

$1\ kJ\ mole^{-1} = 0.2389\ kcal\ mol^{-1} = 0.010363\ eV\ atom^{-1}$

$1\ kcal\ mole^{-1} = 4.1840\ kJ\ mol^{-1} = 0.043360\ eV\ atom^{-1}$

$1\ eV\ atom^{-1} = 96.490\ kJ\ mol^{-1} = 23.062\ kcal\ mol^{-1}$

$1\ ft\ lb = 1.356\ J$ $\quad$ $1\ BTU = 1055\ J$

$1\ erg = 10^{-7}\ J$ $\quad$ $1\ kWh = 3.600 \times 10^6\ J$

FORCE

$1\ N = 0.2248\ lb$ $\quad$ $1\ lb = 4.448\ N$

PRESSURE

$1\ Pa = 1\ N.m^{-2} = 1.45 \times 10^{-4}\ psi = 9.869 \times 10^{-6}\ atm.$

$1\ atm. = 1.013 \times 10^5\ Pa = 1.01325\ bar = 760\ torr\ (mm\ Hg)$

$1\ psi = 6.895 \times 10^3\ Pa$

Print Your Last Name | First Name | Student Number

UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE271.3

Midterm Examination
Part A

Instructor: S.O. Kasap — November 6, 2000
Time allowed: Part A is one hour

Note: Closed book examination. *Answer any 2 questions from 4 questions.* All questions carry equal marks. Marks for part-questions depend on the relative difficulty. All answers must be given in conventional units. All sketches must be clearly labeled and self-explanatory. Diagrams that are not properly and clearly labeled and are subject to ambiguity will be heavily penalized. State clearly all assumptions made in your derivations.

You must hand in this exam before you can start on Part B. All answers must be written in this book

Question	Marks
1	25
2	
3	~~16~~ 15
4	
TOTAL	~~41~~ /50

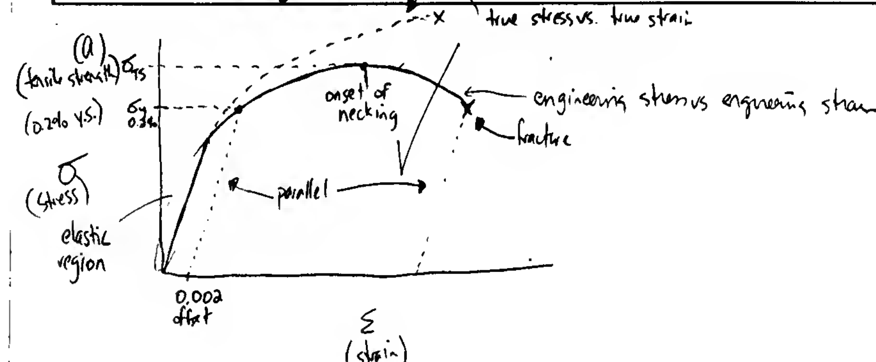
Very good
SOK.

40/50 SOK.

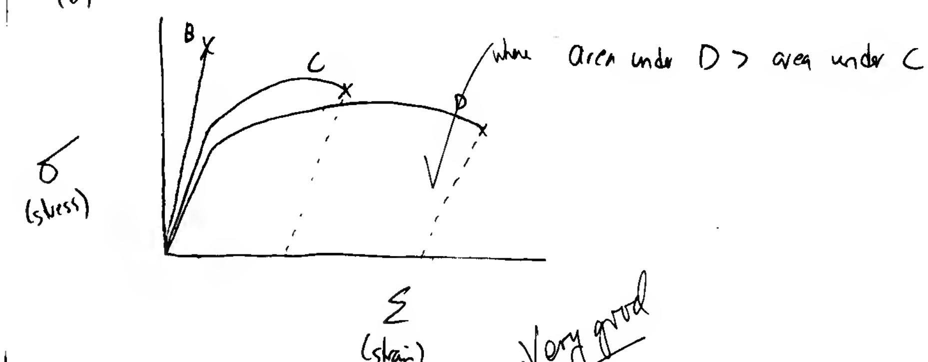
Print Your Last Name | First Name | Student Number

1. (a) Sketch schematically the engineering stress vs. engineering strain, and true stress vs. true strain behavior of an aluminum alloy specimen under a tension test (use the same $\sigma-\varepsilon$ axes for both). Assume the specimen is eventually fractured. Identify on your sketches, the elastic region, 0.2% offset yield strength, plastic deformation region, tensile strength, the onset of necking, and fracture.
(b) Sketch schematically, all on the same graph, the stress vs. strain behavior of three different metals labeled as B, C and D that have the following properties.
B is brittle but C and D are ductile.
B is the strongest (highest strength).
D is more ductile than C
C is stronger than D
D is tougher than C.

(a)



(b)



Very good

UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE271.3

Midterm Examination
Part B

Instructor: S.O. Kasap
Time allowed: Part B is one hour

November 7, 2001

Note: Open book examination. *Answer any 2 questions from 3 questions.* All questions carry equal marks. Marks for part–questions are shown in [ ] in the left hand margin. All answers must be given in conventional units. State clearly all assumptions made in your derivations. All answers must be given in conventional units. Method of solution must be clearly shown. Numerical mistakes, incorrect, unconventional or missing units will be heavily penalized. Mention the source of materials data used. Unless otherwise stated all % are in weight % (w/o).

You must hand in Part A before you can start Part B. Write your answers in the university answer book.

[9] **1.** (a) Aluminum (Al) is a metal with the FCC crystal structure. The radius of the Al atom is 0.143 nm. The atomic mass of the Al is atom 27.00 amu (g mol^{-1}). Calculate the number of Al atoms per unit volume and density of Al. Calculate the atomic packing factor of the Al crystal as %.

[6] (b) Calculate the specific heat capacity of Al

[10] (c) The energy required to create a vacancy in the Al crystal is 0.79 eV. Calculate the concentration of vacancies in the Al crystal at 660 °C; near its melting temperature.

[10] **2.** (a) Consider a 6 meter steel wire of diameter 3 mm that is carrying a tensile load of 1250 N (equivalent to a mass of about 125 kg). If the elastic modulus and Poisson's ratio of steel are 210 GPa and 0.27 what is the new length and diameter of the steel wire assuming that the elongation is elastic? Compare the true and engineering strains and also true and engineering stresses.

[15] (b) A tension test is carried out on an Al alloy specimen which has an original diameter of 12.80 mm. After fracture, the diameter at the fractured neck is measured to be 9.10 mm. The original length between two points on the specimen (between gauge marks) is 5.00 cm. After the fracture, this length is 5.95 cm. The maximum load during the test was 40 kN whereas at fracture the load was 33 kN.

(i) What is the tensile strength?
(ii) What is the true stress at fracture?
(iii) What is the true strain at fracture?
(iv) What is the ductility of the sample?

[25] **3.** (a) X-diffraction experiments carried out on a cubic metal crystal using an X-ray beam with a wavelength l = 0.1542 nm (CuKα emission) give the first five diffraction angles listed in the table below. What is the crystal structure and the lattice parameter?

Observed diffraction peaks from an unknown metal cubic crystal.

Peak (first, second etc.)	1	2	3	4	5
Diffraction angle, 2θ	44.6°	52.0°	76.5°	93.2°	98.8°

(b) Suppose that the x-ray wavelength is λ = 0.07110 nm (MoKα emission). What would be the diffraction angles? Which wavelength would be more adavantageous in studying the x-ray diffraction and the crystal structure?

PHYSICAL CONSTANTS AND USEFUL INFORMATION

$c = 2.9979 \times 10^8$ m s^{-1}
$e = 1.6021 \times 10^{-19}$ C
$m_e = 9.1091 \times 10^{-31}$ kg
$h = 6.62608 \times 10^{-34}$ J s
$\hbar = h/(2\pi) = 1.05459 \times 10^{-31}$ J s

$N_A = 6.0221 \times 10^{23}$ mol^{-1}
$k = 1.3807 \times 10^{-23}$ J K^{-1}
$\varepsilon_o = 8.8542 \times 10^{-12}$ F m^{-1}
$\mu_o = 4\pi \times 10^{-7}$ H m^{-1}

Gas constant, $R = N_A k$ = 8.3144 J K^{-1} mol^{-1} = 0.083144 L.bar K^{-1} mol^{-1}
Mass of proton = 1.67495×10^{-27} kg
Mass of hydrogen atom = 1.6736×10^{-27} kg
Acceleration due to gravity (at 45° latitude), g=9.81 m s^{-2}

SOME CONVERSION FACTORS

LENGTH
1m = 39.37in = 3.280ft = 6.2137×10^{-4} miles
1 in = 0.0254m

ENERGY
1 kJ mole^{-1} = 0.2389 kcal mol^{-1} = 0.010363 eV atom^{-1}
1 kcal mole^{-1} = 4.1840 kJ mol^{-1} = 0.043360 eV atom^{-1}
1 eV atom^{-1} = 96.490 kJ mol^{-1} = 23.062 kcal mol^{-1}
1 ft lb = 1.356 J   1 BTU = 1055 J
1 erg = 10^{-7} J   1 kWh = 3.600×10^6 J

FORCE
1 N =0.2248 lb   1 lb = 4.448 N

PRESSURE
1 Pa =1 N.m^{-2} =1.45×10^{-4} psi = 9.869×10^{-6} atm.
1 atm. = 1.013×10^5 Pa = 1.01325 bar = 760 torr (mm Hg)
1 psi = 6.895×10^3 Pa

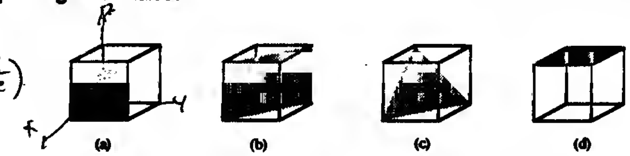
1. (a) Sketch schematically the engineering stress vs. engineering strain, and true stress vs. true strain behavior of an aluminum alloy specimen under a tension test (use the same σ–ε axes for both). Assume the specimen is eventually fractured. Identify on your sketches, the elastic region, 0.2% offset yield strength, plastic deformation region, tensile strength, the onset of necking, and fracture.
(b) Sketch schematically, all on the same graph, the stress vs. strain behavior of three different metals labeled as B, C and D that have the following properties.

- B is brittle but C and D are ductile.
- B is the strongest (highest strength).
- D is more ductile than C
- C is stronger than D
- D is tougher than C.

2. (a) Figure Q.2 (a) to (d) show atomic planes in a cubic crystal. For each of the planes find the corresponding miller indices.

Miller Indicies $= \left(\frac{1}{x} \quad \frac{1}{1} \quad \frac{1}{z}\right)$

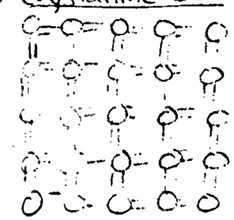
where x, y, z are the axis intercepts



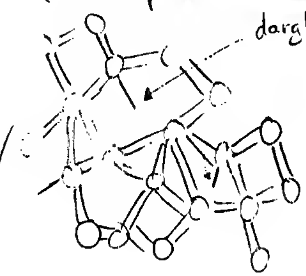
(1 0 0) (1 1 0) (1 1 1) (0 0 1)

(b) Sketch schematically in two-dimensions the structure of
(i) Crystalline Si
(ii) a-Si
(iii) a-Si:H

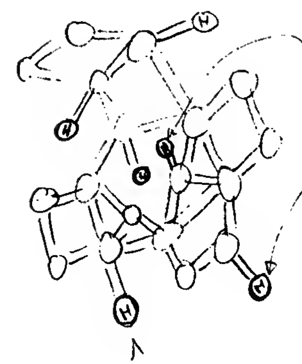
(i) Crystalline Si



(ii) amorphous Silicon



(iii) amorphous Silicon: hydrogen

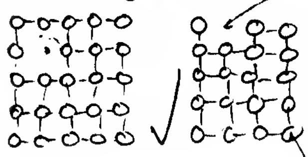


25

3. Sketch schematically the following in a crystal
(a) A vacancy. What is the significance of vacancies?
(b) A large substitutional impurity and a small substitutional impurity.
(c) An interstitial impurity.
(d) Sketch schematically an edge dislocation in a crystal.
(e) Sketch schematically Schottky and Frenkel defects in an ionic crystal (e.g. NaCl)
(f) Two possible (but distinctly different) ionized substitutional impurity defects in an ionic crystal.

(a) Vacancies: missing element from structure

vacancy



vacancy

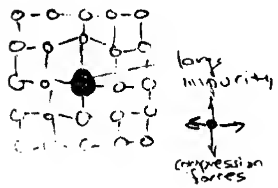
crystal structure

individual atom

Vacancies:
- Vacancies allow for impurities to travel/diffuse into the crystal structure
- Vacancies change the electro-conductivity of the crystal. Usually traps electrons.
- can be used to create electronic devices
- ~~# of vacancies~~ is proportional to temperature

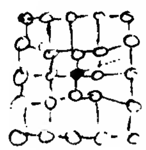
exp?

(b)



large impurity

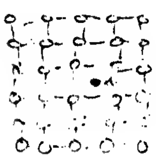
compression forces



small impurities

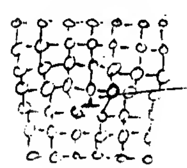
tension forces

(c)



interstitial impurity (not bonded)

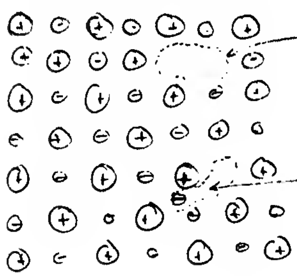
(d)



edge dislocation

NaCl crystal $Na^+ = \oplus$ $Cl^- = \ominus$

e)

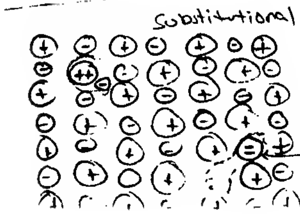


Schottky defect (missing pair)

Frenkel defect (relocated atom creating a vacancy)

f)

substitutional impurity



substitutional impurity

4 Sketch schematically the following
(a) The number of atoms per unit energy vs. energy for gas atoms (e.g. neon gas) in a cylinder at two temperatures T_1 and $T_2 > T_1$. What is the average energy? Identify how you would find the number of atoms with energies greater than E_A? (assume E_A >> average energy)
(b) What is the Maxwell principle of equipartition of energy? Using this principle derive the Dulong-Petit rule.
(c) Sketch schematically, in two dimensions, the surface of a Si crystal that has not been passivated to show the types of surface defects that exist on the surface of a crystal.

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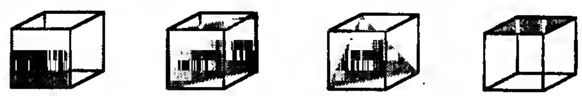
2. (a) Figure Q.2 (a) to (d) show atomic planes in a cubic crystal. For each of the planes find the corresponding miller indices.



(a) (b) (c) (d)

(b) Sketch schematically in two-dimensions the structure of
   (i) Crystalline Si
   (ii) a-Si
   (iii) a-Si:H

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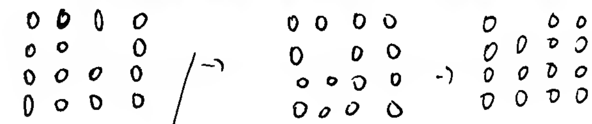
3. Sketch schematically the following in a crystal
   (a) A vacancy. What is the significance of vacancies?
   (b) A large substitutional impurity and a small substitutional impurity.
   (c) An interstitial impurity.
   (d) Sketch schematically an edge dislocation in a crystal.
   (e) Sketch schematically Schottky and Frenkel defects in an ionic crystal (e.g. NaCl)
   (f) Two possible (but distinctly different) ionized substitutional impurity defects in an ionic crystal.

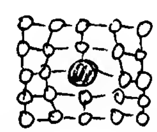
(a)



- cause thermodynamic changes (from movement)
- not stable
- Significance

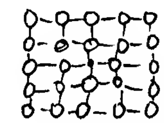
-1

(b) large sub. impurity



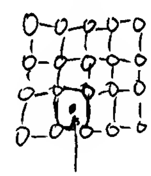
pushes other particles

small sub impurity

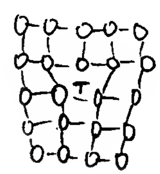


pulls other particles

(c)



(d)



16/25

(e) -4

UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE271.3

Midterm Examination
Part B

Instructor: S.O. Kasap
Time allowed: Part B is one hour

November 6, 2000

Note: Open book examination. *Answer any 2 questions from 3 questions.* All questions carry equal marks. Marks for part–questions are shown in [ ] in the left hand margin. All answers must be given in conventional units. State clearly all assumptions made in your derivations. All answers must be given in conventional units. Method of solution must be clearly shown. Numerical mistakes, incorrect, unconventional or missing units will be heavily penalized. Mention the source of materials data used. Unless otherwise stated all % are in weight % (w/o).

You must hand in Part A before you can start Part B. Write your answers in the university answer book.

[9] **1.** (a) Niobium (Nb) is a metal with the BCC crystal structure. The radius of the Nb atom is 0.143 nm. The atomic mass of the Nb is atom 92.91 amu (g mol^{-1}). Calculate the number of Nb atoms per unit volume and density of Nb. What is the atomic packing factor of the Nb crystal
[5] (b) Calculate the specific heat capacity of Nb.
[11] (c) Suppose that x-diffraction experiments are carried out on this Nb crystal using an x-ray beam with a wavelength $\lambda = 0.07110$ nm (MoK_α emission). What are the first four diffraction angles and what are the corresponding crystal planes?

12] **2.** (a) Consider a 5 meter steel wire of diameter 2.5 mm that is carrying a tensile load of 1 kN (equivalent to a mass of about 100 g). If the elastic modulus and Poisson's ratio of steel are 210 GPa and 0.27 what is the new length and diameter of the steel wire assuming that the elongation is elastic? Compare the true and engineering strains and also true and engineering stresses.
13] (b) A tension test is carried out on an Al alloy specimen which has an original diameter of 12.8 mm. After fracture, the diameter at the fractured neck is measured to be 9.6 mm. The original length between two points on the specimen (between gauge marks) is 5 cm. After the fracture, this length is 5.6 cm. The maximum load during the test was 42 kN whereas at fracture the load was 37 kN.
(i) What is the tensile strength?
(ii) What is the true stress at fracture?
(iii) What is the true strain at fracture?

10] **3.** (a) Energy required to form a vacancy in copper is 0.9 eV/atom. Density of copper is 8.4 g cm^{-3} and its atomic weight is 63.5 g/mole. What is the number of vacancies per cm^{-3} in the met At 1100 °C?
15] (b) For a 75% cold worked aluminum alloy, the recrystallization time is about 200 hr. at 250 and only 1hr. 30min. at 300 °C. At what temperature would the recrystallization process occur i min?
[Note: For information only, the melting temperature of Al is 660 °C]

PHYSICAL CONSTANTS AND USEFUL INFORMATION

$c = 2.9979 \times 10^8$ m s^{-1} — $N_A = 6.0221 \times 10^{23}$ mol^{-1}
$e = 1.6021 \times 10^{-19}$ C — $k = 1.3807 \times 10^{-23}$ J K^{-1}
$m_e = 9.1091 \times 10^{-31}$ kg — $\varepsilon_o = 8.8542 \times 10^{-12}$ F m^{-1}
$h = 6.62608 \times 10^{-34}$ J s — $\mu_o = 4\pi \times 10^{-7}$ H m^{-1}
$\hbar = h/(2\pi) = 1.05459 \times 10^{-31}$ J s

Gas constant, $R = N_A k = 8.3144$ J K^{-1} mol^{-1} = 0.083144 L.bar K mol^{-1}
Mass of proton = 1.67495×10^{-27} kg
Mass of hydrogen atom = 1.6736×10^{-27} kg
Acceleration due to gravity (at 45° latitude), g=9.81 m s^{-2}

UNITS

SI UNITS		
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
DERIVED SI UNITS		
Electric charge	coulomb	C = A s
Electrical resistance	ohm	Ω = V/A = kg m^2 A^{-2} s^{-3}
Electrical conductance	siemen	S = 1/Ω
Electrical capacitance	farad	F = A s V^{-1} = A^2 s^4 kg^{-1} m^{-2}
Electrical inductance	henry	H = V s A^{-1} = kg m^2 s^{-1} A^{-2}
Energy	joule	J = kg m^2 s^{-2} = N m
Force	newton	N = kg m s^{-2}
Magnetic flux	weber	Wb = V s = kg m^2 A^{-1} s^{-2}
Magnetic flux density	tesla	T = Wb m^{-2} = V s m^{-2} = kg A^{-1} s^{-2}
Pressure	pascal	Pa=N m^{-2}
Power	watt	W=J s^{-1}=kg m^2 s^{-3}
Electric potential difference	volt	V = N m C^{-1} = kg m^2 s^{-3} A^{-1}
Frequency	hertz	Hz = s^{-1}

Your Last Name First Name Student Number

UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE217.3

Midterm Examination :
Part A

Instructor: S.O. Kasap
Time allowed: Part A is one hour

November 8, 1999

Note: Closed book examination. *Answer any 2 questions from 4 questions.* All questions carry equal marks. All answers must be given in conventional units. All sketches must be clearly labeled and self-explanatory. Diagrams that are not properly and clearly labeled and are subject to ambiguity will be heavily penalized. State clearly all assumptions made in your derivations. Unless otherwise stated all % are in weight % (w/o).

You must hand in this exam before you can start on Part B. All answers must be written in this book

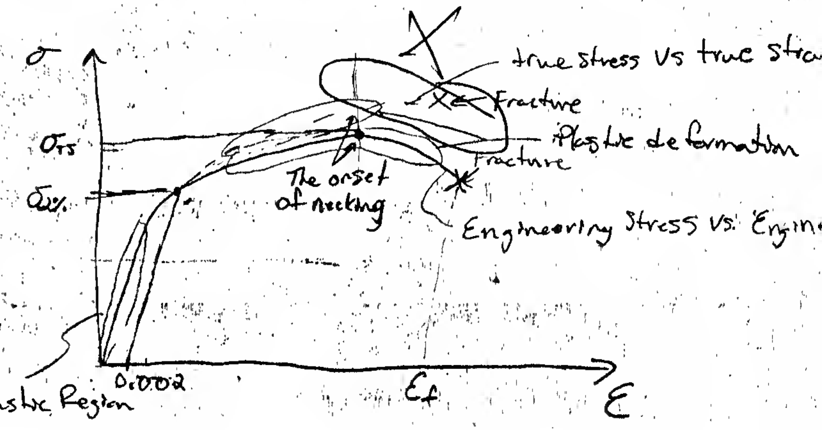
Question	Marks
1	21
2	25
3	
4	
TOTAL	46/50

80%

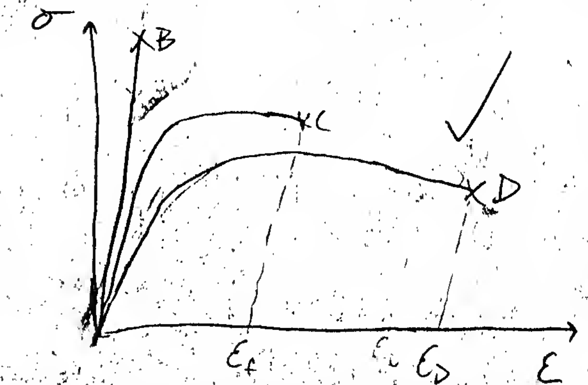
Your Last Name First Name Student Num

1. (a) Sketch schematically the engineering stress vs. engineering strain, and true stress vs behavior of an aluminum alloy specimen under a tension test (use the same σ–ε axes fo Assume the specimen is eventually fractured. Identify on your sketches, ~~the elastic regio~~ ~~offset yield strength, plastic deformation region~~, tensile strength, ~~the onset of necking~~, and
(b) Sketch schematically, all on the same graph, the stress vs strain behavior of three dif metals labeled as B, C and D that have the following properties.
~~B is brittle but C and D are ductile.~~
~~B is the strongest (highest strength).~~
~~D is more ductile than C~~
~~C is stronger than D~~
~~D is tougher than C.~~

a)



b)

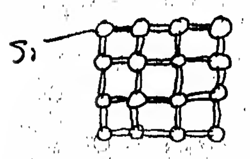


21/25

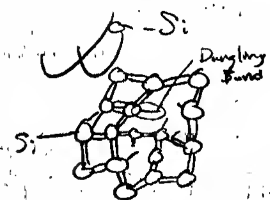
Your Last Name First Name Student Number

2. (a) Sketch schematically in two-dimensions the structure of
(i) Crystalline Si
(ii) a-Si
(iii) a-Si:H
(b) Sketch schematically in two dimensions, the structure of a typical grain boundary surface showing the various defects that exist at the grain boundary. How are grain boundaries formed? How do they affect mechanical properties?

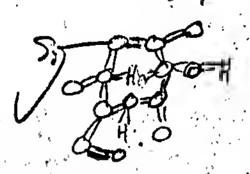
a)



- perodicy
- long range order

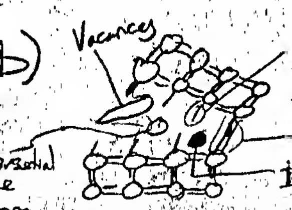


- Short Range order
- Some dangling Bonds
- irregular bond angles



- irregular bond angles
- Hydrogen bonded to what were dangling bonds

b)



- grain boundaries are formed due to crystalization in different areas in the liquid. The nuceli that are formed are at different orientations, and when they grow to grains the boundaries are at different angles. Also caused by rapid cooling.

- grain boundaries affect mechanical properites by increase the strength of the material. this is because dislocations ~~too~~ can't travel from one grain to the next very easily. The dislocation would require more ~~energy~~, energy, thus a greater stress. An increase in grain boundaries yields

Your Last Name First Name Student Number

3. Sketch schematically the following in a crystal
(a) A vacancy.
(b) A large substitutional impurity and a small substitutional impurity.
(c) An Interstitial impurity.
(d) What is the significance of vacancies?
(e) Sketch schematically an edge dislocation in a crystal.
(f) What are typical energy values in electronvolts to create a point vacancy and a dislocation per nanometer of dislocation length.
(g) What is the significance of dislocations (edge and screw)?

Your Last Name [redacted] First Name [redacted] Student Number [redacted]

4. sketch schematically the following
(a) (Molar heat capacity/3R) vs. (Temperature/Debye Temperature). What is the significance of this plot?
(b) The number of atoms per unit energy vs. energy for gas molcules in a cylinder at two temperatures T_1 and $T_2 > T_1$. What is the average energy? Identify how you would find the number of atoms with energies greater than E_A? (assume E_A >> average energy)
(c) (i) Sketch the cooling curve of pure copper as it cools from the molten state to room temperature. Identify all the relevant parameters and the phases that exist.
(ii) Sketch the cooling curve of a Cu-30%Ni alloy as it cools from the molten state to room temperature. Identify all the relevant parameters and the phases that exist.

ANSWERS TO NUMERICAL CALCULATIONS
EE271 - Final - 2000 Exam

All assumptions must be stated stated.

---

1. Below 24.5K, Ne is a crystalline solid with an FCC structure. The interatomic interaction energy per Ne atom can be written as

$$E(r) = -2\varepsilon\left[14.45\left(\frac{\sigma}{r}\right)^6 - 12.13\left(\frac{\sigma}{r}\right)^{12}\right];\ \text{eV/atom}$$

where ε and σ are *constants* particular for a given material and crystal system. For crystalline Ne, $\varepsilon = 3.121\times10^{-3}$ eV and $\sigma = 0.274$ nm.

[4] (a) Using the above relationship, find the equilibrium interatomic separation in the Ne crystal?
[4] (b) Find the bonding energy per atom in solid Ne.
[5] (c) Calculate the density of solid Ne (Atomic Mass of Ne = 20.18 amu).
[4] (d) Estimate the elastic modulus of solid Ne.

Differentiate the E(r) curve, set it to zero ar $r = r_o$, and obtain $\mathbf{r_o = 1.0902\sigma = 0.299}$ **nm**. This is the equilibrium separation; use this r_o in E(r) to find the bond energy $\mathbf{E_o = 0.027}$**eV per atom**. r_o is nearest separation of 2 atoms, so that $2R = r_o$. From R find **a = 0.4224 nm**. **Density = 1.86 g cm^{-3}**. Bulk modulus, $K = E''/(9r_o)$, where E'' is the second differential of E(r). **K ≈ 1.29 GPa. Elastic modulus, Y will be of the same order as K.**

---

[5] 2. (a) A copper wire that has a 1 mm diameter and is carrying a current of 5 A. Calculate the joule heating per unit volume, as W mm^{-3}, and the drift velocity of the electrons. How does this drift velocity compare with the mean speed of the electrons? What is the electric field in the sample?

A = 7.85E-7 m^2; J = 6.37E6 A m^{-2}; $P = \rho J^2$ = **6.9E5 W m^{-3} or 0.69 mW mm^{-3}**; n = 8.45E28 m^{-3}; $J = \sigma E$ gives E = 0.108 V/m; $J = qv_d$ gives $\mathbf{v_d = 0.00047}$ **m/s**

[12] (b) Constantan has the composition 45%Ni - 55%Cu. Cu-Ni alloys show complete solid solubility. As an alloy constantan is widely used in resistor applications (up to 500 °C), in strain gauges as well as one of the thermocouple metal pairs. Calculate the resistivity (ρ), TCR (α) and thermal conductivity (κ) of constantan and compare the values with the experimental measurements, ρ(20 °C) = 5×10^{-7} Ω m, TCR (20 °C) = 2×10^{-5} K^{-1}, κ = 21 W m^{-1} K^{-1} What are the reasons for the differences in the calculated and experimental values?

ρ_{alloy} = **326 nΩ m**; κ_{alloy} = **22.5 W m^{-1} K^{-1}**; α_{alloy} = **0.00021 1/K**

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[17] 3. A device engineer fabricates a sensor by depositing an a-Si:H film on a glass substrate and then depositing an Al electrode onto the a-Si:H film. The device has the structure glass/a-Si:H/metal. The glass substrate is 5 mm thick. The a-Si: is 2 μm thick and the Al electrode is 0.1 μm thick. The expansion coefficients of glass, a-Si:H and Al are about 7×10^{-6} K^{-1}, 4×10^{-6} K^{-1}, 23×10^{-6} K^{-1} respectively. Calculate the stress in each layer if the temperature drops from 25 °C to –50 °C. State your assumptions and for each stress identify whether it is tensile or compressive.

The whole composite system is in equilibrium. Let A = Glass substrate, B - Silicon film, C = Metal electrode. Let a = thickness of A, b = thickness of B, c = thickness of C. Let w = width, common to all, A, B, C. Stress in A is σ_A. Force acting on A is $\sigma_A$wa.

Net force = $\sigma_A(wa) + \sigma_B(wb) + \sigma_C(wc) = 0$ (1)

For each write the total strain

$\varepsilon_A = \lambda_A\Delta T + \sigma_A/E_A$ (2)

$\varepsilon_B = \lambda_B\Delta T + \sigma_B/E_B$

$\varepsilon_C = \lambda_C\Delta T + \sigma_C/E_C$ (3)

All layers are glued together: $\varepsilon_A = \varepsilon_B = \varepsilon_C$. (4)

Solve equations (1) to (4) simulatenously (they are linear equations) to obtain σ_A, σ_B, and σ_C. Note, w cancels out.

Use λ and E values take from the Thermal Expansion chapter and Mechanical Properties 1 chapter.

Let A = Glass; B = a-Si:H semiconductor film; C = Al electrode

a = 5E-6 m, b = 2E-6 m; c = 1E-7 mm.

λ_A = 7E-6 1/K; λ_B = 4E-6 1/K; λ_C = 23E-6 1/K

E_A = 70 GPa; E_B = 100 GPa; E_C = 70 GPa

Solving Eqs (1) to (4) we find:

Glass: σ_A = 7.3E4 = 7.3 kPa (tensile)

a-Si:H film: σ_B = -22.5 MPa (compressive)

Al electrode: σ_C = 84.0 MPa (tensile)

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[17] 4. Consider a wafer of Si crystal which has a uniform boron (B) doping of 1×10^{17} cm^{-3} in the bulk. Suppose that the Si wafer is exposed to a phosphorus (P) gas at 1250 °C for 10 minutes. During the diffusion process, the surface P concentration remains saturates at about 1×10^{21} cm^{-3}. Where is the junction from the surface? How long does it take to have the same junction depth from the surface if the diffusion temperature is 1100 °C? State the assumprions used in your calculations.

The diffusion coefficient of P in Si has D_o = 3.85 cm^2 s^{-1}, and E_d = 3.66 eV/atom.
Note: B doping makes Si, *p*-type and P-doping makes it *n*-type. The junction is where B and P concentrations are equal.

113
125-126

UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE217.3

Final Examination
Part B

Instructor: S.O. Kasap — December 10, 1999
Time allowed: Part B is $1^1/_2$ hour

Note: Open book examination. Calculators are allowed. *Answer any 3 questions from 5 questions.* If you answer more than 3 questions, only the first three will be marked. All questions carry equal marks. All answers must be given in conventional units. State clearly all assumptions made in your derivations. Method of solution must be clearly shown. Numerical mistakes, incorrect, unconventional or missing units will be heavily penalized. Mention the source of materials data used. Unless otherwise stated all % are in weight % (^{w}/o).

Important: *You must hand in Part A before you can start Part B.* Write your answers in the university answer book.

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[7] 1. (a) InAs is a III-V semiconductor that has the zinc blende structure with a lattice parameter of 0.606 nm. It is widely used in optoelectronic devices. Calculate the atomic concentration and the density of InAs.

[10] (c) Iron (< 912 °C) has the BCC crystal with a lattice parameter a = 0.287 nm. Find the planar concentrations (atoms per unit area of a plane) as number of atoms per nm^2 of the {100}, {110} and {111} planes. Which plane has the most concentration of atoms per unit area?

---

2. An 80 at % Cu–20 at % Zn brass disk of 40 mm diameter and 5 mm thickness is used to conduct heat from a heat source to a heat sink.

[5] (a) Calculate the thermal resistance of the brass disk.

[5] (b) If the disk is conducting heat at a rate of 100 W, calculate the temperature drop along the disk.

[7] (c) What should be the composition of brass if the temperature drop across the disk is to be halved?

---

3. A *strain gauge* is a transducer attached to a body to measure its fractional elongation $\Delta L/L$ under an applied load (force) F. The gauge is a grid of many folded runs of a thin, resistive wire glued to a flexible backing, as depicted in Figure 2.29 in the textbook. The gauge is attached to the body under test such that the resistive wire length is parallel to the strain.

[5] (a) Assume that the elongation does not change the resistivity and show that the change in the resistance ΔR is related to the strain, $\varepsilon = \Delta L/L$ by

$$\Delta R \approx R(1+2\upsilon)\varepsilon$$

where υ is the Poisson ratio, which is defined by

$$\nu = -\frac{\text{Transverse strain}}{\text{Longitudinal strain}} = -\frac{\varepsilon_t}{\varepsilon_l}$$

where ε_l is the strain along the applied load, that is, $\varepsilon_l = \Delta L/L = \varepsilon$, and ε_t is the strain in the transverse direction, that is, $\varepsilon_t = \Delta D/D$, where D is the diameter (thickness) of the wire.

[2] (b) Explain why a nichrome wire would be a better choice than copper for the strain gauge (consider the TCR).

[4] (c) How do temperature changes affect the response of the gauge? Consider the effect of temperature on ρ. Also consider the differential expansion of the specimen with respect to the gauge wire such that even if there is no applied load, there is still strain, which is determined by the differential expansion coefficient,

15.3

15.364695

$\lambda_{specimen} - \lambda_{gauge}$, where λ is the thermal coefficient of linear expansion: $L = L_0[1 + \lambda(T - T_0)]$, where T_0 is th reference temperature.

[2] (d) The gauge factor for a transducer is defined as the fractional change in the measured property $\Delta R/R$ pe unit input signal (ε). What is the gauge factor for a metal-wire strain gauge, given that for most metals, $\upsilon \approx$ 1/3?

[4] (e) Consider a strain gauge that consists of a nichrome wire of resistivity 1 $\mu\Omega$ m, a total length of 1 m, an a diameter of 25 μm. What is ΔR for a strain of 10^{-3}? Assume that $\upsilon \approx 1/3$.

---

4. (a) Show that if R_H is the Hall coefficient and σ is the conductivity of a metal then the drift mobility of electrons is given by

$$\mu_d = |\sigma R_H|$$

(b) An n-type Si sample doped with arsenic has the rectangular dimensions of length=1 cm, width=0.1 cn and thickness=1 mm. A constant current of 100 mA is passed through the sample along its length. When it is placed in a magnetic field of 0.1 T acting along its width, a Hall voltage of 500 µV is measured. If the voltage drop along the sample is 434 mV calculate the drift mobility of electrons in doped Si.

[In n-type Si, the electron concentration n>> the hole concentration p, so that you can safely neglect the few holes in the sample and assume that only electrons contribute to the electric current]

---

[7] 5. (a) Amorphous selenium (a-Se) is a high resistivity semiconductor that has a density of approximatel 4.3 g cm^{-3} and an atomic number and mass of 34 and 78.96 g/mol. Its relative permittivity at 1 kHz has bee measured to be 6.7. Calculate the relative magnitude of the local field in a-Se. Calculate the polarizability per Se atom in the structure. What type of polarization is this? How will ε_r depend on the frequency?

[10] (a) Consider a CsBr crystal that has the CsCl unit cell crystal structure (one Cs$^+$–Br$^-$ pair per unit cell) with a lattice parameter (a) of 0.430 nm. The electronic polarizability of Cs$^+$ and Br$^-$ ions are 3.35 × 10^{-40} F m^2 and 4.5 × 10^{-40} F m^2 respectively, and the mean ionic polarizability per ion pair is 5.8 × 10^{-40} F m^2. Wha is the low frequency dielectric constant and that at optical frequencies?

PHYSICAL CONSTANTS AND USEFUL INFORMATION

$c = 2.9979 \times 10^8$ m s^{-1}	$N_A = 6.0221 \times 10^{23}$ mol^{-1}
$e = 1.6021 \times 10^{-19}$ C	$k = 1.3807 \times 10^{-23}$ J K^{-1}
$m_e = 9.1091 \times 10^{-31}$ kg	$\varepsilon_o = 8.8542 \times 10^{-12}$ F m^{-1}
$h = 6.62608 \times 10^{-34}$ J s	$\mu_o = 4\pi \times 10^{-7}$ H m^{-1}
$\hbar = h/(2\pi) = 1.05459 \times 10^{-31}$ J s	

Gas constant, R = $N_A k$ = 8.3144 J K^{-1} mol^{-1} = 0.083144 L.bar K^{-1} mol^{-1}
Mass of proton = 1.67495 × 10^{-27} kg
Mass of hydrogen atom = 1.6736 × 10^{-27} kg
Acceleration due to gravity (at 45° latitude), g =9.81 m s^{-2}

UNITS

SI UNITS		
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
DERIVED SI UNITS		
Electric charge	coulomb	C = A s
Electrical resistance	ohm	Ω = V/A = kg m^2 A^{-2} s^{-3}

UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE217.3

Midterm Examination
Part B

Instructor: S.O. Kasap | November 8, 1998
Time allowed: Part B is one hour

Note: Open book examination. *Answer any 2 questions from 3 questions.* All questions carry equal marks. Marks for part–questions are shown in [ ] in the left hand margin. All answers must be given in conventional units. State clearly all assumptions made in your derivations. All answers must be given in conventional units. Method of solution must be clearly shown. Numerical mistakes, incorrect, unconventional or missing units will be heavily penalized. Mention the source of materials data used. Unless otherwise stated all % are in weight % (w/o).

You must hand in Part A before you can start Part B. Write your answers in the university answer book.

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[9] **1.** (a) Platinum has the FCC crystal structure. The radius of the Pt atom is 0.1386 nm. The atomic mass of Pt is 195.09 amu (g mol^{-1}). Calculate the number of Pt atoms per unit volume and density of Pt. What is the atomic packing factor of the Pt crystal APF = .74 4 21.5 g cm

[5] (b) Calculate the specific heat capacity of Pt. c = .128 J g^{-1} K^{-1}

[11] (c) Suppose that x-diffraction experiments are carried out on this Pt crystal using an x-ray beam with a wavelength λ = ~~0.7110 n~~m (MoK_α emission). What are the first four diffraction angles and what are the corresponding crystal planes?

Plane 1 111
2 200
3 220
4 311

$\lambda = 0.0711$ nm

12] **2.** (a) A tension test is carried out on an Al alloy specimen which has an original diameter of 0.505 in. After fracture, the diameter at the fractured neck is measured to be 0.41 in and the gauge length after fracture is 2.22 in (original gauge length was 2in). The maximum load during the test was 40 kN whereas at fracture the load was 36 kN.

(i) What is the tensile strength? σ_{TS} 310 MPa
(ii) What is the true stress at fracture? σ_{True} = 423 MPa
(iii) What is the true strain at fracture? ε_T = .104

13] (b) Consider an optical fiber (a glass fiber) with an "invisible" crack on the surface that has a depth of 0.3 μm and a radius of curvature of 0.2 nm. The diamater of the optical fiber is 75 μm. Given that the elastic modulus of glass is 70 GPa, its surface tension is 1 J m^{-2} what is the maximum load this fiber can carry before it fractures?

σ_{cb} = 385.4 MPa

---

[25] **3.** A device engineers fabricates a metal-semiconductor diode. The metal is a thin film of Al (not necessarily pure Al) on a Si crystal wafer. Suppose that the Al film is 0.1 mm thick and the Si crystal wafer is 100 mm thick. Estimate the temperature induced strain in the Al film if the device is taken from ~~25 °C~~ 25 C down to the liquid nitrogen temperature 77 K (-196 °C). What is the stress if the deformation were elastic? The yield strength of this Al-alloy is 50MPa. What is the mechanical strain that results in yield (plastic deformation)? What is the mechanical strain in the Al film? Do you think the deformation of the Al thin film is elastic? Estimate the temperature at which the Al film begins to deform plastically (reaches the yield point)?

Over the temperature range of interest (from -196 to 25 °C), aluminum has a thermal expansion coefficient of about 10×10^{-6} K^{-1} and silicon has a thermal expansion coefficient of about 1×10^{-6} K^{-1}. The elastic modulus of Al is 70 Gpa.

PHYSICAL CONSTANTS AND USEFUL INFORMATION

c = 2.9979 × 10^8 m s^{-1} | N_A = 6.0221 × 10^{23} mol^{-1}
e = 1.6021 × 10^{-19} C | k = 1.3807 × 10^{-23} J K^{-1}
m_e = 9.1091 × 10^{-31} kg | ε_o = 8.8542 × 10^{-12} F m^{-1}
h = 6.62608 × 10^{-34} J s | μ_o = 4π × 10^{-7} H m^{-1}
$\hbar$ = $h/(2\pi)$ = 1.05459 × 10^{-31} J s

Gas constant, R = $N_A k$ = 8.3144 J K^{-1} mol^{-1} = 0.083144 L.bar K^{-1} mol^{-1}
Mass of proton = 1.67495 × 10^{-27} kg
Mass of hydrogen atom = 1.6736 × 10^{-27} kg
Acceleration due to gravity (at 45° latitude), g =9.81 m s^{-2}

UNITS

SI UNITS		
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
DERIVED SI UNITS		
Electric charge	coulomb	C = A s
Electrical resistance	ohm	Ω = V/A = kg m^2 A^{-2} s^{-3}
Electrical conductance	siemen	S =1/Ω
Electrical capacitance	farad	F = A s V^{-1} = A^2 s^4 kg^{-1} m^{-2}
Electrical inductance	henry	H = V s A^{-1} = kg m^2 s^{-1} A^{-2}
Energy	joule	J = kg m^2 s^{-2} = N m
Force	newton	N = kg m s^{-2}
Magnetic flux	weber	Wb = V s = kg m^2 A^{-1} s^{-2}
Magnetic flux density	tesla	T = Wb m^{-2} = V s m^{-2} = kg A^{-1} s^{-2}
Pressure	pascal	Pa=N m^{-2}
Power	watt	W=J s^{-1}=kg m^2 s^{-3}
Electric potential difference	volt	V = N m C^{-1} = kg m^2 s^{-3} A^{-1}
Frequency	hertz	Hz = s^{-1}

EE 217 / 317 / 487 FORMULA SHEET

Notation

<>	average
<v>	average v
ab/cd	$(ab)/(cd) = \dfrac{ab}{cd}$
F	vector F

All symbols have their usual meanings within context.

Kinetic Molecular Theory and Rates

$PV = (N/N_A)RT$ — N = number of molecules, R = gas constant

$P = {}^1/_3 nm\langle v^2\rangle = {}^1/_3\rho\langle v^2\rangle$ — ρ = density, n = number of molecules per unit volume

$\langle KE\rangle = {}^1/_2 m\langle v^2\rangle = {}^3/_2 kT$ — Monatomic gas with translational KE only

$U = N_A({}^1/_2 m\langle v^2\rangle) = (3/2)N_A kT$ — Internal energy per mole for monatomic gas

$C_V = [\partial U/\partial T]_V = {}^3/_2 kN_A = {}^3/_2 R$ — Monatomic gas

$C_V = [\partial U/\partial T]_V = {}^1/_2 fkN_A = {}^1/_2 fR$ — f = Degree of Freedom

$C_V = [\partial U/\partial T]_V = 3R = 25\text{J K}^{-1}\text{ mole}^{-1}$ — Solids-metals

$C_P \approx C_V \approx 3N_A k = 3R = 25\text{J K}^{-1}\text{ mole}^{-1}$ — Solids-metals

$$n_v(v) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right)$$ Concentration of molecules per unit velocity

$$n_E = n_v(v)\left(\frac{dv}{dE}\right)$$ Concentration of molecules per unit energy

$$n_E(E) = \frac{2}{\sqrt{\pi}} N\left(\frac{1}{kT}\right)^{3/2} E^{1/2} \exp\left(-\frac{E}{kT}\right)$$ Concentration of molecules per unit energy

$$\frac{n_E(E)}{N} = C\exp\left(-\frac{E}{kT}\right)$$ Boltzmann distribution

$(\Delta x)_{rms} = \sqrt{\dfrac{kT}{K}}$ K = spring constant

$v_{rms} = [4kTRB]^{1/2}$ — R = resistance, B = bandwidth

$\upsilon = A\upsilon_o \exp(-E_A/kT)$; $E_A = U_{A*} - U_A$ — Rate of transformation

Crystallography and Elementary Materials Science

$n = \rho N_A/M_{at}$

n = atomic concentration (cm^{-3}), M_{at} = atomic mass (g/mole), ρ = density (g cm^{-3})

SOME CONVERSION FACTORS

LENGTH

1m = 39.37in = 3.280ft = 6.2137×10^{-4}miles

1 in = 0.0254m

ENERGY

1 kJ mole^{-1} = 0.2389 kcal mol^{-1} = 0.010363 eV atom^{-1}

1 kcal mole^{-1} = 4.1840 kJ mol^{-1} = 0.043360 eV atom^{-1}

1 eV atom^{-1} = 96.490 kJ mol^{-1} = 23.062 kcal mol^{-1}

1 ft lb = 1.356 J — 1 BTU = 1055 J

1 erg = 10^{-7} J — 1 kWh = 3.600×10^6 J

FORCE

1 N = 0.2248 lb — 1 lb = 4.448 N

PRESSURE

1 Pa = 1 N.m^2 = 1.45×10^{-4} psi = 9.869×10^{-6} atm.

1 atm. = 1.013×10^5 Pa = 1.01325 bar = 760 torr (mm Hg)

1 psi = 6.895×10^3 Pa

$a = 4R/\sqrt{3}$ BCC

$a = 4R/\sqrt{2}$ FCC

$\ell = \frac{1}{n^{1/3}}$ Mean separation ℓ between impurity atoms given their atomic concentration n

$n_s = n^{2/3}$ Surface concentration and bulk concentration

$n_v = N_o \exp\left(-\frac{Q_v}{RT}\right)$ Vacancy concentration

Diffraction Theory

$2d\sin\theta = n\lambda$

$d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}}$ Separation between planes $(h\,k\,l)$

Conditions for x-ray diffraction peaks

FCC: h, k, l are all odd numbers or all even numbers; no mixing of odd and even numbers

BCC: $h + k + l$ = even number.

Phase Diagrams

$W_L + W_S = 1$ [L = Liquid, S = Solid] and $W_L = \frac{C_S - C_O}{C_S - C_L}$

Bonding

$U(r) = -\frac{A}{r^n} + \frac{B}{r^m}$ General expression for the interatomic net PE

$S_o = \left[\frac{d^2U}{dr^2}\right]_{r=r_o}$ Spring constant

$F = S_o(r-r_o)$ Definition of spring constant

$E = S_o/r_o$ or $E = \frac{1}{r_o}\left[\frac{d^2U}{dr^2}\right]_{r_o}$ Elastic modulus E

$p = Qx$ $F = p\frac{dE}{dr}$ $E(r) = \frac{2p}{4\pi\varepsilon_o r^3}$

$p_{induced} = \alpha E$ $F = p_{induced}\left[\frac{dE}{dr}\right] \sim \frac{\alpha}{r^3}\left[\frac{d}{dr}\left(\frac{1}{r^3}\right)\right] \sim \frac{\alpha}{r^7}$

$F = p_o\left[\frac{dE}{dr}\right] - p_o\left[\frac{d}{dr}\left(\frac{1}{r^3}\right)\right] \sim \frac{p_o^2}{r^4}$

Diffusion and oxidation

$\Gamma = \frac{\Delta N}{(\Delta A)(\Delta t)}$ Definition of particle flux

$\Gamma = -D\frac{\partial C}{\partial x}$ Fick's first law. C = concentration

$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$ Fick's second law

$D = D_o \exp\left[-\frac{Q_d}{RT}\right]$ Diffusion coefficient. Q_d = activation energy for diffusion.

$\frac{(C_x - C_o)}{(C_s - C_o)} = 1 - \mathrm{erf}\left[\frac{x}{2\sqrt{Dt}}\right]$ Unlimited supply of dopants

$C_x = \frac{N_o}{\sqrt{\pi Dt}}\exp\left[-\frac{x^2}{4Dt}\right]$ Constant number of dopants

N_o is the total number of tracer atoms per unit area deposited on the surface at $t = 0$.

$x = \frac{A}{2}\left[\left(1 + \frac{t+\tau}{A^2/4B}\right)^{1/2} - 1\right]$ SiO_2 growth kinetics

$t = \frac{1}{B}x^2 + \frac{A}{B}x - \tau$

$\frac{B}{A} = K_1\exp(-\frac{E_1}{kT})$; $B = K_2\exp(-\frac{E_2}{kT})$

$\tau = \frac{x_i^2 + Ax_i}{B}$

Mechanical Properties

$\sigma = E\varepsilon$ $\tau = G\gamma$ $p = -K\Delta$ $\upsilon = -\varepsilon_t/\varepsilon_l$

$E = 2G(1+\upsilon)$ $K = \frac{E}{3(1-2\upsilon)}$

$\varepsilon_t = \ln(L/L_o) = \ln(\varepsilon+1)$ True Strain

$\tau(\theta) = (\sigma/2)\sin(2\theta)$ Shear stress and applied tensile stress

TS = 3.45(BHM) Brinell Hardness and Tensile Strength

$H_V \approx 3\sigma_y$ Vickers hardness

$\varepsilon_{CW} = \Delta A/A$ Cold work

$\sigma_y = \sigma_o + Kd^{-1/2}$ Yield strength and grain size

$R = \sigma_{min}/\sigma_{max}$ Ratio of minimum to maximum stress for a cyclic applied stress

$PD = 2St$ [Pipeline]

UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE217.3

Midterm Examination
Part B

Instructor: S.O. Kasap
Time allowed: Part B is one hour

November 5, 1998

Note: Open book examination. *Answer any 2 questions from 3 questions*. All questions carry equal marks. Marks for part–questions are shown in [ ] in the left hand margin. All answers must be given in conventional units. State clearly all assumptions made in your derivations. All answers must be given in conventional units. Method of solution must be clearly shown. Numerical mistakes, incorrect, unconventional or missing units will be heavily penalized. Mention the source of materials data used. Unless otherwise stated all % are in weight % (w/o).

You must hand in Part A before you can start Part B. Write your answers in the university answer book.

1. Silver has the FCC crystal structure. The radius of the Ag atom is 0.1444 nm. The relative atomic mass ("atomic weight") of Ag is 106.4 (g mol^{-1})

[4] (a) Calculate the lattice parameter, a, of the silver crystal. $a = 0.4084$ nm
[4] (b) Calculate the number of atoms per unit volume (in the units of cm^{-3}). 5.872×10^{22} cm^{-3}
[4] (c) Calculate the atomic packing factor (APF) in the crystal. 0.74
[4] (d) Calculate the density of silver in g cm^{-3}. 10.38 g/cm^3
[4] (e) Calculate the specific heat capacity (as heat capacity per unit gram) of silver. $C_m \sim 0.235$ J/Kg
[5] Check (f) Calculate the planar concentration of atoms (number of atoms per unit area) on the planes (100) and (110) [express your answer in number of atoms (nm)$^{-2}$].
11.99 Atoms/nm^2 8.479 Atoms/nm^2

[12] **2.** (a) An aluminum alloy (30003-H14) rod which is 10 mm in diameter and 1 m in length is subjected to a 6 kN tensile load. What is the new length and diameter of the rod, in mm, given that the Young's modulus for this alloy is 70 GPA, yield strength is 145 MPa and the Poisson ratio $\upsilon = 0.33$. What is the elastic energy stored per unit volume?

[13] (b) A tension test is carried out on an Al alloy specimen which has an original diameter of 0.505 in. After fracture, the diameter at the fractured neck is measured to be 0.37 in and the gauge length after fracture is 2.32 in (original gauge length was 2in). The maximum load during the test was 37 kN whereas at fracture the load was 34 kN.

(i) What is the tensile strength?
(ii) What is the true stress at fracture?
(iii) What is the true strain at fracture?

[25] **3.** X-diffraction experiments using x-rays with a wavelength $\lambda = 0.07107$ nm on an unkown cubic crystal has given the following diffraction angles (2θ),

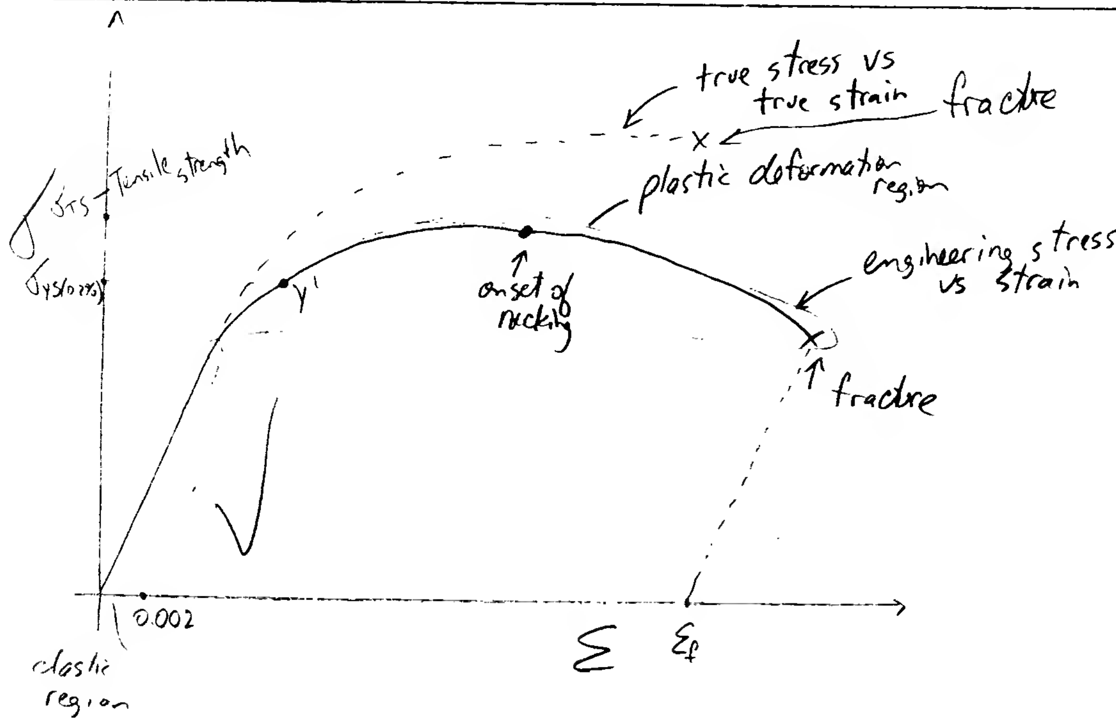
Peak	First Peak	Second Peak	Third Peak	Fourth Peak
Diffraction angle	17.5°	24.9°	30.6°	35.4°

Find whether this is a simple, body centered or face centered cubic crystal (SC, BCC or FCC) and determine the indices of the planes producing the diffraction. Determine the lattice parameter *a*.

$a = 0.33$ nm

(110) (200) (211) (220)

1. Sketch schematically the engineering stress vs engineering strain, and true stress vs. true strain behavior of an aluminum alloy specimen under a tension test (use the same σ–ε axes for both). Assume the specimen is eventually fractured. Identify on your sketches, the elastic region, 0.2% offset yield strength, plastic deformation region, tensile strength, the onset of necking, and fracture.



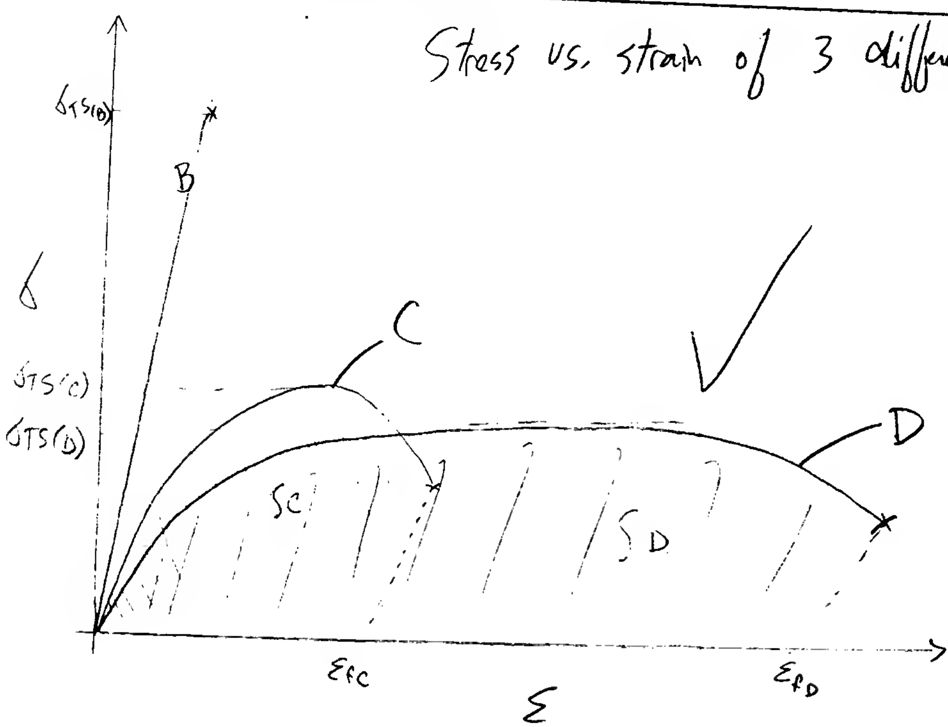
Stress vs. strain in aluminum alloy (a ductile material)

10

2. Sketch schematically, all on the same graph, the stress vs strain behavior of three different metals labeled as B, C and D that have the following properties.

B is brittle but C and D are ductile.
B is the strongest (highest strength).
D is more ductile than C
C is stronger than D
D is tougher than C.

Stress vs. strain of 3 different metals.



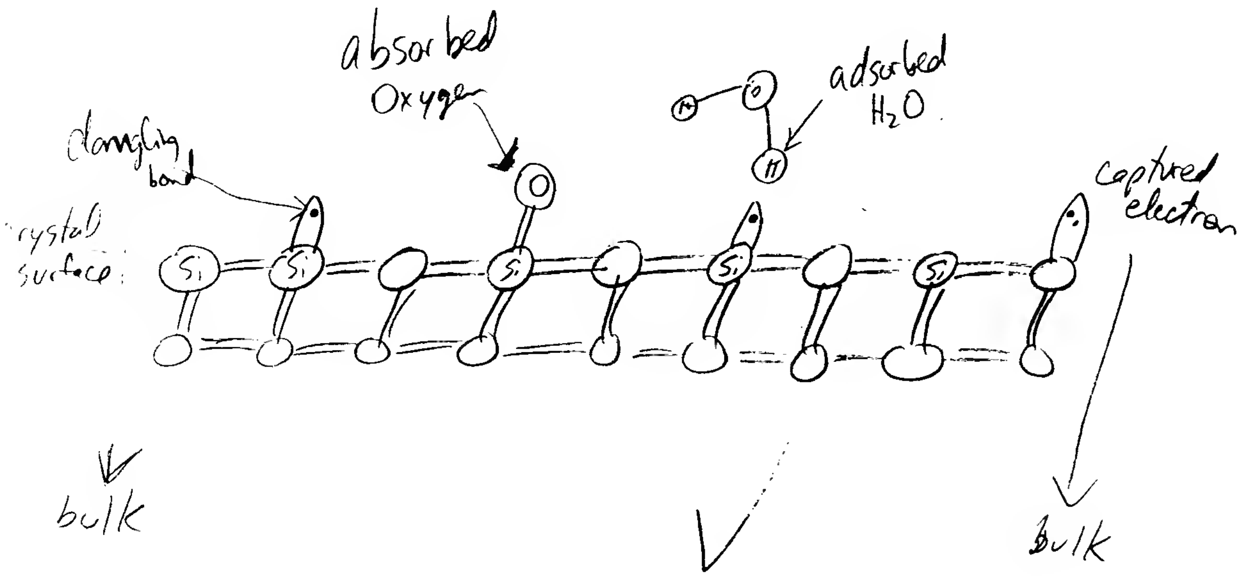
$\sigma_{TS(B)} > \sigma_{TS(C)} > \sigma_{TS(D)}$ Strength

$S_D > S_C$ Toughness

$\varepsilon_{fC} < \varepsilon_{fD}$ Ductility

10

4. Sketch schematically, in two dimensions, the surface of a Si crystal that has not been passivated to show the types of surface defects that exist on the surface of a crystal. Explain the difference between surface absorption and adsorption.

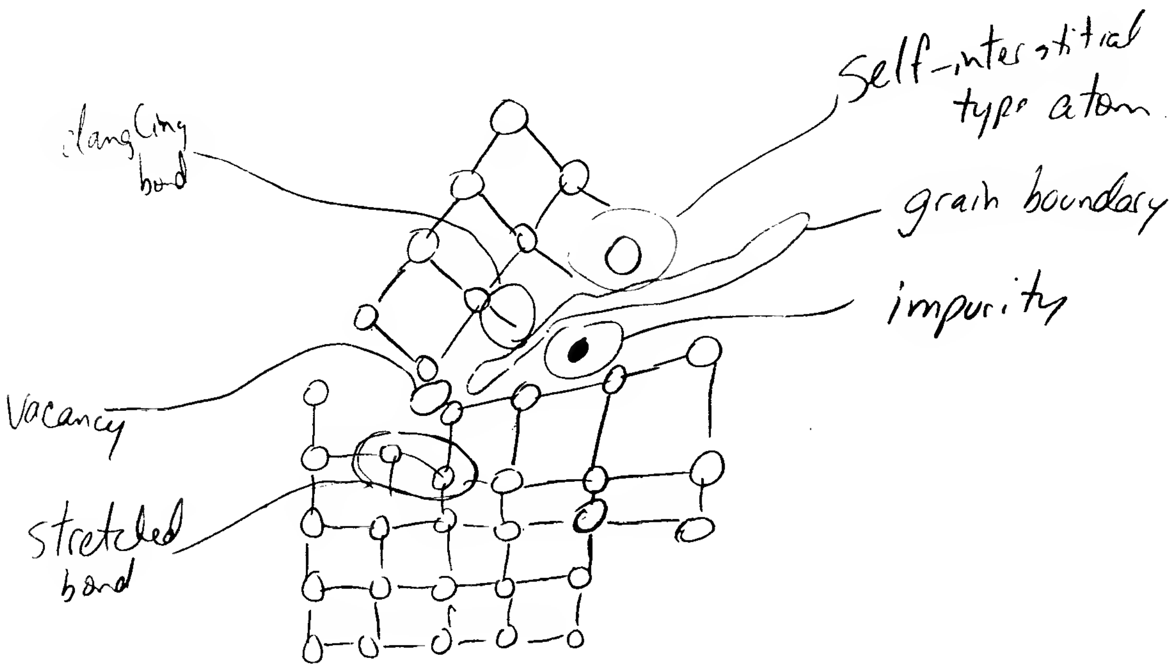


Surface absorption occurs when another compound bonds by primary bonding to the surface of the crystal surface

Surface adsorption occurs when another compound (generally a polar compound) bonds by secondary bonding (Van der Waal's, dipole forces) to the surface of the crystal surface

10.

5. Sketch schematically in two dimensions, the structure of a typical grain boundary surface showing the various defects that exist at the grain boundary. How are grain boundaries formed? How do they affect mechanical properties?



Grain boundaries are formed by crystallization. When a liquid crystallizes, it first nucleates and forms small pockets of single crystals. When these crystals grow, they interact with each other and form boundaries. This results in grain boundaries at different orientations.

These boundaries are significant in mechanical properties as we deal with dislocations. When a dislocation strikes a grain boundary, it is much more difficult to move. This results in a larger yield stress. In the same way, an increase in grain boundaries (smaller grains) results in a higher yield stress. They also affect thermal and electrical conductivity, since it is more difficult to conduct heat or electrons through a void. One must also watch for slippage along the voids left in a grain boundary.

10

UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE217.3
Final
~~Midterm~~ Examination
Part B

Instructor: S.O. Kasap — December 12, 1998
Time allowed: Part B is $1^1/_2$ hour

Note: Open book examination. Calculators are allowed. *Answer any 3 questions from 6 questions.* If you answer more than 3 questions, only the first three will be marked. All questions carry equal marks. All answers must be given in conventional units. State clearly all assumptions made in your derivations. Method of solution must be clearly shown. Numerical mistakes, incorrect, unconventional or missing units will be heavily penalized. Mention the source of materials data used. Unless otherwise stated all % are in weight % ($^w/o$).

Important: *You must hand in Part A before you can start Part B.* Write your answers in the university answer book.

---

[7] **1.** (a) Calculate the specific heat capacity of a 62 wt.% Sn and 38 wt. %Pb solder given the atomic masses 118.7 and 207.2 g mol^{-1} for Sn and Pb respectively.

[10] (b) A cold worked copper component takes 45 minutes to recrystallize at 120 °C but takes 6 minutes at 170 °C. How long will the recrystallization take place at 370 °C. Can you cold work or strain harden this component at 370 °C? Why can't you cold work Pb and Sn at room temperature?

---

2. Consider the 50%Pb-50%Sn solder.

[4] (a) Sketch the temperature-time profile and the microstructure of the alloy at various stages as it is cooled from the melt.

[1] (b) At what temperature does the solid melt? 270 °C

[2] (c) What is the temperature range over which the alloy is a mixture of melt and solid? What is the structure of the solid?

[10] (d) Consider the 50% Pb-50% Sn solder at room temperature following cooling from above 183°C. Assume that the rate of cooling from 182°C down to room temperature is faster than the rates of atomic diffusion in the solid to change the compositions of the α and β phases. If the mass of the whole alloy is 1kg, calculate the masses of the following components in the solid.

(i) The primary α — 0.277 kg
(ii) α in the whole alloy
(iii) α in the eutectic solid
(iv) β in the alloy (where is the β-phase?)

---

3 Electron drift mobility in tin (Sn) has been measured to be 3.9 cm^2 V^{-1} s^{-1}. The room temperature (20°C) resistivity of Sn is 12×10^{-8} Ω m and TCR (α_o) at 0°C is 4.63×10^{-3} K^{-1}. Relative atomic mass (M_{at}) and density of Sn are 118.69 and 7.30 g cm^{-3} respectively.

[5] (a) How many "free" electrons are donated by each Sn atom in the crystal? How does this compare with the position of Sn in Group IVB of the Periodic Table?

[4] (b) Calculate the thermal conductivity if Sn at 20 °C and at 50 °C.

[6] (c) For a particular device design, the temperature dependence of the resistivity of Sn is to be represented

$$\rho = \rho_o(T/T_o)^n$$

where n is a characteristic index and ρ_o is the resistivity at the reference temperature T_o.

Given the temperature coefficient of resistivity TCR (α_o) for Sn at 0°C, what should n be for $T_o = 0$°C. Why do you think n is not unity?

[2] (d) What is the TCR at 20 °C?

Note: Use only data for Sn in this question and no other data; otherwise the answer is wrong.

---

[5] **4.** (a) The density of CsCl is 4 g cm^{-3}. The atomic masses of Cs and Cl are 132.9 g mol^{-1} and 35.45 g mol^{-1} respectively. Calculate the lattice parameter of CsCl. 0.412 nm.

[12] (b) Consider a CsCl crystal which has the CsCl unit cell crystal structure (one Cs^+–Cl^- pair per unit cell) with a lattice parameter (a) calculated in in Part (a). The electronic polarizability of Cs^+ and Cl^- ions are 3.35×10^{-40} F m^{-2}, and 3.40×10^{-40} F m^{-2} respectively and the mean ionic polarizability per ion pair per is 6×10^{-40} F m^{-2}. What is the low frequency dielectric constant and that at optical frequencies?

Hint: At low frequencies both ionic and electronic polarization contribute to ε_r whereas at high frequencies (optical frequencies) only the electronic polarization contributes to ε_r because the ions are too sluggish to be displaced quickly by the electric field. Only the light electrons can be displaced at very high (optical) frequencies.

$\varepsilon_r = 7.58$ 2.712.

---

[6] **5.** (a) Show that the thermal expansion coefficient for an area is 2λ. Consider an aluminum square sheet of area 1 cm^2. If the thermal expansion coefficient of Al at room temperature (25 °C) is about 24×10^{-6} K^{-1}, at what temperature is the percentage change in the area is +1%

[11] (b) A device engineers fabricates a metal-semiconductor diode. The metal is a thin film of an Al-Si alloy (Al alloyed with a few percentage Si) on a Si crystal wafer. Suppose that the Al alloy film is 0.1 μm thick and the Si crystal wafer is 100 μm thick. Estimate the temperature induced strain in the Al-alloy film if the device is taken from 25 °C down to the liquid nitrogen temperature 77 K (−196 °C). What is the stress if the deformation were elastic? Do you think this deformation is elastic if the yield strength is about 50 MPa? Estimate the temperature at which the deformation becomes plastic?

NOTE: Assume over the temperature range of interest,

$$E_{Al} \approx E_{Si} \approx 70 \text{ GPa},\ \lambda_{Al} \approx 10\times10^{-6}\ \text{K}^{-1} \text{ and } \lambda_{Si} \approx 1\times10^{-6}\ \text{K}^{-1}.$$

---

[17] **6.** A HV co-axial cable has an Al inner conductor of diameter 8 mm and a polyethylene (PE) dielectric insulation of thickness 4 mm. The outer conductor is of copper. The cable is expected to carry a dc voltage of 100 kV. If the cable is laid under water, assuming a maximum ambient temperature of 25°C, evaluate the maximum dc current that can be safely conducted through the cable. Assume a safety factor of 3 between the maximum electric field in the dielectric and the breakdown field of the insulator. Electrical resistivity of Al is 27 nΩ m at 25 °C. See Table 2.1, p. 113 for ρ_o and α_o. Thermal conductivity of polyethylene is 0.3 W m^{-1} K^{-1}. The breakdown field of the PE insulator at 25°C is 50 MV m^{-1}. The general temperature dependence of the breakdown field for PE is given in Figure 7.51, p. 592 and see Figure Q.5.

(b) At what dc current value the cable is guaranteed to breakdown ?

(c) What are these current values if the safety factor is 1?

Note: Safety factor is E_b/E_{max}

Note: All assumptions must be stated.

UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE217.3

Final Examination

Instructor: S.O. Kasap
Time allowed: Three hours

December 9, 1997

Note: Open book examination. Only the course textbook and the formulae sheet are allowed. No other notes are allowed. You can use calculators. *Answer any 6 questions from 9 questions.* All questions carry equal marks. Marks for a part–question is proprotional to the difficulty of the question. All answers must be given in conventional units. State clearly all assumptions made in your derivations and reference the origin of any materials data (e.g. Table and page numbers in the course textbook). All answers must be given in conventional units. Method of solution must be clearly shown. Numerical mistakes, incorrect, unconventional or missing units will be heavily penalized. Unless otherwise stated all % are in weight % (w/o).

1. In general, the potential energy per atom or per ion in an ionic crystal can be written as the sum of an attractive PE and a repulsive PE,

$$E(r) = -\frac{A}{r^n} + \frac{B}{r^m} \qquad (1)$$

where A and n are constants characterizing the attractive PE and B and m are constants characterizing the repulsive PE. The bulk modulus K is related to the energy curve. In its simplest form (assuming a simple cubic unit cell) K can be estimated from

$$K = \frac{1}{9r_o}\left[\frac{d^2E}{dr^2}\right]_{r=r_o}$$

The CsCl has the following bond energy characteristics: $A = (1.763e^2)/(4\pi\varepsilon_o)$ where e is the electronic charge and ε_o is the absolute permittivity, $n = 1$, $m = 9$ and the equilibrium separation $r_o = 0.357$ nm. Calculate the bond energy in eV per atom and Joules per mole, and estimate the bulk modulus and Young's modulus given $\upsilon = 0.25$.

2. (a) Germanium has the diamond crystal structure. The density of Ge is 5.32 g cm^{-3} and its atomic mass is 72.60 g mol^{-1}. Find the lattice parameter a.

(b) KCl has the same crystal structure as NaCl. The lattice parameter a of KCl is 0.629 nm. The atomic masses of K and Cl are 39.10 g mol^{-1} and 35.45 g mol^{-1} respectively. Calculate the density of KCl.

(c) Explain very briefly why the elastic modulus should depend on the crystal direction.

(d) Sketch schematically the elastic modulus of iron as a function of temperature passing through the BCC to FCC transformation at about 912°C.

(e) Calculate the specific heat capacity of a 62 wt.% Sn and 38 wt. %Pb solder given the atomic masses 118.7 and 207.2 g mol^{-1} for Sn and Pb respectively.

3. (a) Sketch schematically the engineering stress vs. engineering strain, and true stress vs true strain behavior of an aluminum alloy specimen under a tension test. Assume the specimen is eventually fractured. Identify on your sketches, the elastic region, 0.2% offset yield strength, plastic deformation region, tensile strength, the onset of necking, and fracture.

(b) (i) Sketch schematically a typical creep curve of strain vs. time from the instant of application of a constant stress σ_1 at a temperature T_1 well above $0.4T_m$, where T_m is the melting temperature. Identify and label on your sketch the various important regions. Sketch also the creep curves at a higher temperature $T_2 > T_1$, and at a higher stress $\sigma_2 > \sigma_1$ (but at the same temperature T_1).

(ii) Sketch schematically log(stress) vs log(rupture time) from creep-rupture test at two temperatures well above 0.4Tm.

(c) Consider a metal specimen subjected to a cyclic stress. Sketch schematically typical fatigue curves of stress amplitude vs log(number of cycles to failure) for an aluminum alloy and a low carbon steel. Identify the endurance limit in your sketch.

(d) Consider the Charpy impact test. Sketch schematically the impact energy vs. temperature behavior for BCC and FCC metals.

4. Consider the 60%Pb-40%Sn solder.

(a) Sketch the temperature-time profile and the microstructure of the alloy at various stages as it is cooled from the melt.

(b) At what temperature does the solid melt?

(c) What is the temperature range over which the alloy is a mixture of melt and solid? What is the structure of the solid?

(d) Consider the 60% Pb-40% Sn solder at room temperature following cooling from above 183°C. Assume that the rate of cooling from 182°C down to room temperature is faster than the rates of atomic diffusion in the solid to change the compositions of the α and β phases. If the mass of the whole alloy is 1kg, calculate the masses of the following components in the solid.

(i) The primary α

(ii) α in the whole alloy

(iii) α in the eutectic solid

(iv) β in the alloy (where is the β-phase?)

1.

5. (a) The energy for vacancy formation in copper is 0.9 eV/atom. Given that the atomic mass is 63.5 g mol^{-1}, the density at room temperature is 8.4 g cm^{-3} at 1000 °C, what is the concentration of vacancies at 1000 °C.

(b) A cold worked copper component takes 113 minutes to recrytallize at 102 °C but takes 24 minutes at 135 °C. How long will the recrystallization take place at 350 °C. Can you cold work or strain harden this component at 350 °C? Why can't you cold work Pb and Sn at room temperature?

---

6. (a) Define engineering strain and true strain. Obtain a relationship between the two strains.

(b) A strain gauge is a transducer that is attached to a body to measure its fractional elongation, $\Delta L/L$, under an applied load, F (see Fig. 2.29, p. 150). It is a grid of many folded runs of a thin resistive wire glued to a flexible backing which is attached to the body under test so that the resistive wire length is parallel to the strain. By assuming that the elongation does not change the resistivity, show that the change in the resistance, ΔR, is related to the strain, $\varepsilon = \Delta L/L$, by

$$\Delta R \approx R(1+2\upsilon)\varepsilon$$

where υ is the Poisson ratio. Note: Consider differentiating

$$R = \frac{\rho L}{\pi\left(\frac{D}{2}\right)^2}$$

or consider a change in R due to a change in L and D.

(c) Gauge factor, F_G, for a transducer is defined as the fractional change in the measured property ($\Delta R/R$) per unit input signal (ε). What is the gauge factor for a metal wire strain gauge given that for most metals $\upsilon \approx 1/3$?

(d) Consider a strain gauge that consists of a nichrome wire of resistivity $10^{-6}\Omega$.m total length of 1m and diameter of 30 μm. What is ΔR for a strain of 5×10^{-3}? Assume that $\upsilon \approx 1/3$.

(e) How does temperature changes affect the response of the gauge? Consider the effect of temperature on ρ and the differential expansion of the specimen under test with respect to the gauge wire so that even if there is no applied load, there is still a strain that is determined by the differential expansion coefficient, $\lambda_{specimen} - \lambda_{gauge}$, where λ is the thermal coefficient of linear expansion; $L = L_0[\,1 + \lambda(T-T_0)\,]$ where T_0 is the reference temperature.

---

7. (a) The electron drift mobility in silver has been measured to be 56 $cm^2\ V^{-1}\ s^{-1}$ at 27 °C. Atomic mass and density of Ag are given as 107.87 (amu or $g.mol^{-1}$) and 10.50 g cm^{-3} respectively. Assuming that each Ag atom contributes one conduction electron, calculate the resistivity of Ag at 27 °C. Compare this value with the measured value of $1.6\times10^{-8}\ \Omega$.m at the same temperature and suggest reasons for the difference.

(b) Consider a CsCl crystal which has the CsCl unit cell crystal structure (one Cs^+–Cl^- pair per unit cell) with a lattice parameter (a) of 0.412 nm. The electronic polarizability of Cs^+ and Cl^- ions are 3.35×10^{-40} F m^2, and 3.40×10^{-40} F m^2 respectively and the mean ionic polarizability per ion pair is 6×10^{-40} F m^2. What is the low frequency dielectric constant and that at optical frequencies? Hint: At low frequencies both ionic and electronic polarization contribute to ε_r whereas at high frequencies (optical frequencies) only the electronic polarization contributes to ε_r.

8. Constantan has the composition 45%Ni-55%Cu. Cu-Ni alloys show complete solid solubility. As an alloy constantan is widely used in resistor applications (up to 500 °C), in strain gauges as well as one of the thermocouple metal pairs. Given that the resistivity and TCR (α_o) of copper at 20 °C are 17 nΩ m and 0.004 K^{-1} respectively and the Nordheim coefficient of Ni dissolved in Cu is 1250 nΩ m, calculate the resistivity (ρ), TCR (α) and thermal conductivity (κ) of constant and compare the values with the experimental measurements, ρ (20 °C) = 5×10^{-7} Ω m, TCR (20 °C) = 2×10^{-5} K^{-1}, κ = 21 W m^{-1} K^{-1} What are the reasons for the differences in the calculated and experimental values?

9. (a) A HV co-axial cable has an Al inner conductor of diameter 10 mm and a polyethylene (PE) dielectric insulation of thickness 3 mm. The outer conductor is of copper. The cable is expected to carry a dc voltage of 40 kV. If the cable is laid under water, assuming a maximum ambient temperature of 25°C, evaluate the maximum dc current that can be safely conducted through the cable. Assume a safety factor of 3 between the maximum electric field in the dielectric and the breakdown field of the insulator. Electrical resistivity of Al is 27 nΩ m at 25 °C. See Table 2.1, p. 113 for ρ_o and α_o. Thermal conductivity of polyethylene is 0.3 W m^{-1} K^{-1}. The breakdown field of the PE insulator at 25°C is 50 MV m^{-1}. The general temperature dependence of the breakdown field for PE is given in Figure 7.51, p. 592 and see Figure Q.9.

(b) At what dc current value the cable is guaranteed to breakdown ?

(c) What are these current values if the safety factor is 1?

[Note: Safety factor is E_{br}/E_{max}]

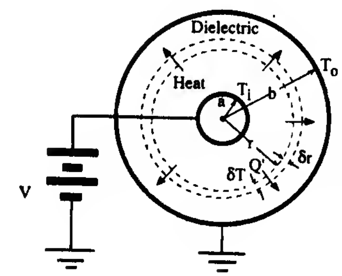
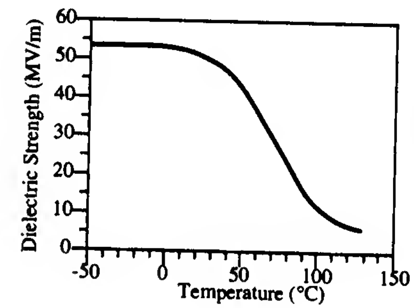


Fig. Q.9: The joule heat generated in the core conductor flows outwards radially through the dielectric material. The dashed concentric circiles identify a thin cylindrical shell concentric with the core.



Typical temperature dependence of the dielectric strenth of a polyethyelene based polymeric insulation.

UNIVERSITY OF SASKATCHEWAN
COLLEGE OF ENGINEERING

ELECTRICAL ENGINEERING EE217.3

Final Examination

Instructor: S.O. Kasap December 13, 1996
Time allowed: Three hours

Note: Closed book examination. *Answer any six questions. Do not answer more than six questions.* All questions carry equal marks. Marks for part–questions are shown in [ ] in the left hand margin. All answers must be given in conventional units. Method of solution must be clearly shown. Numerical mistakes, incorrect, unconventional or missing units, unclear or unlabeled diagrams and sketches will be heavily penalized. Mention the source of materials data used. State clearly all assumptions made in your derivations. Unless otherwise stated all % are in weight % ($^{w}/_{o}$).

---

[6] **1.** (a) Explain, *with clearly labelled diagrams*, the origin of metallic bonding and the important characteristics (material properties) this type of bonding imparts to the solid compared with covalent and ionic bonding.

[11] (b) The energy per ion for the CsCl crystal can written as

$$E(r) = -\frac{Me^2}{4\pi\varepsilon_o r} + B\exp\left(-\frac{r}{\rho_o}\right) \qquad \text{[Joules/ion]} \qquad (1)$$

where r is the interionic separation, M is a dimensionless constant called the Madelung constant of magnitude 1.76267, $B = 4.512\times10^4$ eV and $\rho_o = 0.034$ nm is also a constant. *Note that* exp(x) means e^x. What is the equilibrium separation between the ions in the crystal and the bonding energy (eV) per ion? How does this energy compare with van der Waals bonding?
[Note: You may have to use a graphical solution, or a numerical technique , such as the Newton technique, to solve a non-linear equation. A graph paper is provided if you wish to use a graphical solution]

---

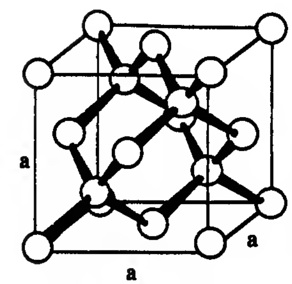
[5] **2.** (a) Given the lattice parameter of Si as a = 0.543 nm, and the atomic mass of Si is 26.09 g.mol^{-1} calculate the number of Si atoms per unit volume in (nm)$^{-3}$ and the density of Si.

[4] (b) Calculate the number of atoms per m^2 and per (nm)^2 on the (100), (110) and (111) planes in the Si crystal. Which plane has the least number of atoms (and hence the least probability of crystal defects on that surface).

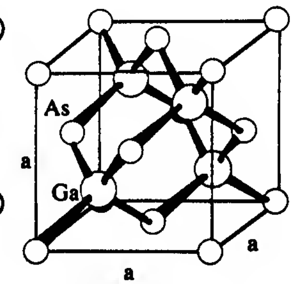
[4] (c) Calculate the atomic vacancy concentration in the Si crystal at room temperature and at 1200°C given that the energy of vacancy formation for Si is 2.4 eV/atom.

[4] (d) GaAs is a semiconductor material which has the zinc blende crystal structure. Given its density (5.318 g.cm^{-3}) and the atomic masses of Ga (69.72 amu) and As (74.92 amu) calculate

the lattice parameter, a, of the unit cell.



Diamond unit cell



Zinc belnde unit cell

---

[7] **3.** (a) Define *flux* and explain Fick's First and Second Laws with examples and clearly labelled diagrams. For Fick's second law give two examples each involving the diffusion of impurity atoms from the surface of a solid into the bulk where one example is for diffusion from a limited source (constant number of impurity atoms in the surface region) and one example for diffusion from an unlimited source (constant concentration of impurity atoms at the surface). Your diagrams should clearly show concentration profiles at various times and have all important features marked. What are the limits of applicability?

[10] (b) Hydrogen can be purified by diffusing the gas through a palladium sheet. In a certain purification process, the concentration of hydrogen, during a steady state operation, at the high and low concentration sides of the Pd sheet have been found to be 2.4 kg per m^3 and 0.6 kg per m^3 of Pd. How many kilograms of hydrogen can be pass through per hour through a 5 mm thick Pd sheet having an area of 0.2 m^2 at 500°C. The diffusion coefficient, D, of hydrogen in Pd has an activation energy, Q_d, of 24.6 kJ.mol^{-1} and a pre-exponential constant, D_o, of 4.5×10^{-7} m.s^{-1}. Mention any assumptions made.

---

[17] **4.** Answer any two from (a) to (c)
(a) Explain, *with clearly labelled diagrams*, the principle of zone-refining and describe with clearly labelled diagrams how zone refined crystals are obtained in industry.
(b) Explain, *with clearly labelled diagrams*, the recrystallization of a cold worked metal specimen. Why can you not cold work tin and lead?
(c) Explain, *with clearly labelled diagrams*, how an oxide grows on a Si wafer and the factors that control the thickness of the oxide. Sketch schematically the time evolution of the oxide thickness at different temperatures and pressures, and for dry and wet oxidation.

---

[8] **5.** (a) Explain, *with clearly labelled diagrams and a practical example*, the Griffith's theory of crack propagation. What are the consequences of the Griffth's theory of crack propagation? Explain why internal and surface cracks are important in glasses and ceramics but not in ductile metals such as copper?

[9] (b) Consider an optical fiber (which is essentially a glass fiber) used in communication

networks. The fiber has a diameter of 100 microns (1 micron = 1 μm = 10^{-6} m). The glass has an elastic modulus (E) of 70 GPa and a surface tension (γ) of about 1 J.m^{-2}. Suppose that the fiber experiences a tensile load (F) of 0.1 N (a typical weight of a baby-bite from an apple; or 10% of an apple). If we assume that the crack tip has a radius of curvature of 0.2 nm, what is the depth (or length) of a surface crack that will lead to the fracture of this fiber?

---

6. Consider the 50%Pb-50%Sn solder.

[3] (a) Sketch the temperature-time profile and the microstructure of the alloy at various stages as it is cooled from the melt.

[1] (b) At what temperature does the solid melt?

[1] (c) What is the temperature range over which the alloy is a mixture of melt and solid? What is the structure of the solid?

[8] (d) Consider the solder at room temperature following cooling from 182°C. Assume that the rate of cooling from 182°C down to room temperature is faster than the rates of atomic diffusion in the solid to change the compositions of the α and β phases. Calculate the masses of the following components in the solid.

(i) The primary α

(ii) α in the whole alloy

(iii) α in the eutectic solid

(iv) β in the alloy (where is the β-phase?)

[4] (e) Calculate the specific heat of the solder given the atomic masses of Pb, 207.2, and that of Sn, 118.71.

---

[8] 7. (a) The electron drift mobility in silver has been measured to be 56 cm^2.V^{-1}.s^{-1} at 27°C. Atomic mass and density of Ag are given as 107.87 and 10.50 g.cm^{-3} respectively. Assuming that each Ag atom contributes one conduction electron, calculate the resistivity of Ag at 27°C. Compare this value with the measured value of 1.6×10^{-8} Ω.m at the same temperature and suggest reasons for the difference.

[9] (b) Electrical engineers frequently take the resistivity of a metal to obey $\rho \propto T^n$ where n is a resistivity index; a constant for each metal and typically greater than unity. For tungsten, ρ = 50 nΩ.m at 0 °C where α_o = 1/202 K^{-1}. Assuming $\rho \propto T^n$ to be valid, find n for tungsten [*Hint:* Consider the definition of α_o).

---

[2] 8. (a) What are the Mathiessens and Noirdheim rules?

[2] (b) Sketch the resistivity and thermal conductivity of Cu as a function of temperature.

[13] (c) An 80 at.%Cu–20 at.%Zn brass disk of diameter 40 mm and thickness 5 mm is to be used to conduct heat from a "heat source" to a "heat sink".

(i) Calculate the thermal resistance of the brass disk.

(ii) If the disk is conducting heat at a rate of 100 W calculate the temperature drop along the disk.

(iii) What should be the composition of brass if the temperatures drop across the disk is to be halved.

NOTE: The room temperature resistivity of Cu is 17 nΩ.m.

---

9. Consider a coaxial cable which has a polyethylene based insulation. The inner conductor is copper with a resistivity of 17 nΩ.m and a diameter of 4 mm. The thickness of the polymer insulation is 3 mm. Various parameters of the coaxial cable are defined in Fig. Q.9. The radii of the inner and outer conductors are a and b respectively.

[7] (a) Suppose that the cable is operating with a constant current, I, through the cable. Suppose that resistivity of of copper at temperature T_o is ρ_o, and that the resistivity obeys $\rho \propto T^n$ where n is an index (approximately valid for the majority of pure metals). For copper n = 1.15. By considering the rate of heat flow Q' through a cylinderical shell of thickness δr, as shown in Fig. Q.9, show that under steady state conditions,

$$\left(\frac{T_o}{T_i}\right)^{n-1} = 1 - \frac{\rho_o I^2 (n-1)}{2\pi^2 a^2 \kappa T_o}\ln\left(\frac{b}{a}\right)$$

where T_o is the temperature of the outer conductor, taken as the ambient temperature, T_i is the temperature of the inner conductor and κ is the thermal conductivity of the insulation. For polyethylene $\kappa \approx$ 0.35 W.m^{-1}.K^{-1} and $\varepsilon_r \approx$ 2.4.

[3] (b) What is the maximum dc operating voltage if the maximum field at room temperature (25°C) must not be greater than $^1/_3\mathcal{E}_{br}$ where $\mathcal{E}_{br}$ is the dielectric strength of the insulation (see dielectric strength graph on page 5).

[4] (c) What is the breakdown current, I_{br}, that will lead to the thermal breakdown of the dielectric insulation when the applied voltage is that in (b)?

[3] (d) What is the percentage increase in the breakdown current if the operating voltage increases by 30%

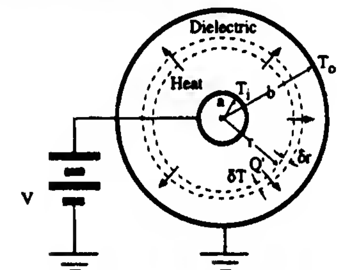
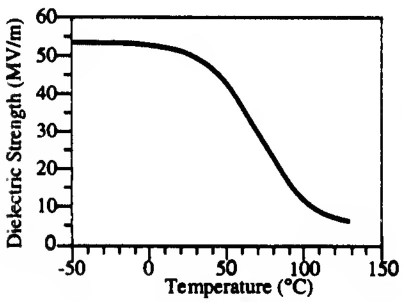


Fig. Q.9: Coaxial power cable



Typical temperature dependence of the dielectric strenth of a polyethyelene based polymeric insulation.

PHYSICAL CONSTANTS AND USEFUL INFORMATION

$c = 2.9979 \times 10^{8}$ m.s^{-1}	$N_A = 6.0221 \times 10^{23}$ mol^{-1}
$e = 1.6021 \times 10^{-19}$ C	$k = 1.3807 \times 10^{-23}$ J.K^{-1}
$m_e = 9.1091 \times 10^{-31}$ kg	$\varepsilon_o = 8.8542 \times 10^{-12}$ F.m^{-1}
$h = 6.62608 \times 10^{-34}$ J.s	$\mu_o = 4\pi \times 10^{-7}$ H.m^{-1}
$\hbar = h/2\pi = 1.05459 \times 10^{-31}$ J.s	

Gas constant, $R = N_A k = 8.3144$ J.K^{-1}.mol^{-1} = 0.083144 L.bar.K^{-1}.mol^{-1}

Mass of proton = 1.67495×10^{-27} kg

Mass of hydrogen atom = 1.6736×10^{-27} kg

Acceleration due to gravity (at 45° latitude), g = 9.81 m.s^{-2}

UNITS

SI UNITS

Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K

DERIVED SI UNITS

Electric charge	coulomb	C = A s
Electrical resistance	ohm	Ω = V/A = kg.m^2.A^{-2}.s^{-3}.
Electrical conductance	siemen	S = 1/Ω
Electrical capacitance	farad	F = A.s.V^{-1} = A^2.s^4.kg^{-1}.m^{-2}.
Electrical inductance	henry	H = V.s.A^{-1} = kg.m^2.s^{-1}.A^{-2}.
Energy	joule	J = kg.m^2.s^{-2} = N.m

Force	newton	N = kg.m.s^{-2}
Magnetic flux	weber	Wb = V.s = kg.m^2.A^{-1}.s^{-2}.
Magnetic flux density	tesla	T = Wb.m^{-2}. = V.s.m^{-2}. = kg.A^{-1}.s^{-2}.
Pressure	pascal	Pa=N.m^{-2}
Power	watt	W=J.s^{-1}=kg.m^2.s^{-3}.
Electric potential difference	volt	V = N.m.C^{-1} = kg.m^2.s^{-3}.A^{-1}
Frequency	hertz	Hz = s^{-1}

SOME CONVERSION FACTORS

LENGTH

1m = 39.37in = 3.280ft = 6.2137×10^{-4} miles

1 in = 0.0254m

ENERGY

1 kJ.mole^{-1} = 0.2389 kcal.mol^{-1} = 0.010363 eV.atom^{-1}

1 kcal.mole^{-1} = 4.1840 kJ.mol^{-1} = 0.043360 eV.atom^{-1}

1 eV.atom^{-1} = 96.490 kJ.mol^{-1} = 23.062 kcal.mol^{-1}.

1 ft.lb = 1.356 J   1 BTU = 1055 J

1 erg = 10^{-7} J   1 kWh = 3.600×10^{6} J

FORCE

1 N = 0.2248 lb   1 lb = 4.448 N

PRESSURE

1 Pa = 1 N.m^2 = 1.45×10^{-4} psi = 9.869×10^{-6} atm.

1 atm. = 1.013×10^{5} Pa = 1.01325 bar = 760 torr (mm Hg)

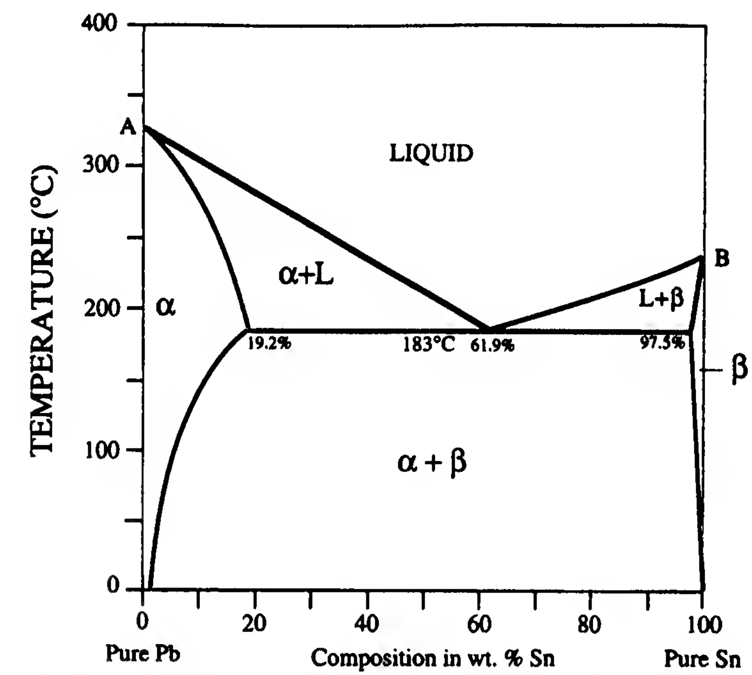
1 psi = 6.895×10^{3} Pa

The erf(z) and erfc(z) Functions
Note: erfc(z) = 1 − erf(z)
Values calculated from Matlab (v. 4.1) for Macintosh

z	Erf(z)	Erfc(z)	z	Erf(z)	Erfc(z)
0	0.00000	1.000000	2.00	0.995 322	4.6777×10^{-3}
0.01	0.00113	0.966 716	2.25	0.996 537	1.4627×10^{-3}
0.05	0.0564	0.943 626	2.50	0.999 593	4.0695×10^{-4}
0.10	0.1125	0.667 537	2.75	0.999 699 376	1.0062×10^{-4}
0.25	0.2763	0.723 673	3.00	0.999 977 909	2.2090×10^{-5}
0.50	0.5205	0.479 500	3.25	0.999 995 697	4.3027×10^{-6}
0.75	0.7112	0.266 644	3.50	0.999 999 256 901	7.4309×10^{-7}
1.00	0.64270	0.157 299	3.75	0.999 999 666 272	1.1372×10^{-7}
1.25	0.92290	0.077 099	4.00	0.999 999 964 562	1.5417×10^{-8}
1.50	0.96610	0.033 894	4.25	0.999 999 996 149	1.6505×10^{-9}
1.75	0.96667	0.013 328	4.50	0.999 999 999 603	1.9661×10^{-10}

Given y_1 at x_1 and y_2 at x_2 and we need y_i at x_i. We use linear interpolation:

$$\frac{y_i - y_1}{x_i - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad y_i = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x_i - x_1)$$



Nordheim Coefficient (at 20°C) for Dilute Alloys of Copper

Solute in Solvent (Element in matrix)	Nordheim Coefficient nΩ.m.	Maximum Solubility at 25°C at. %
Au in Cu matrix	5500	100
Mn in Cu matrix	2900	24
Ni in Cu matrix	1250	100
Sn in Cu matrix	2900	0.8
Zn in Cu matrix	300	30

EE217-1996 Final. Q.1.

Given the following parameters

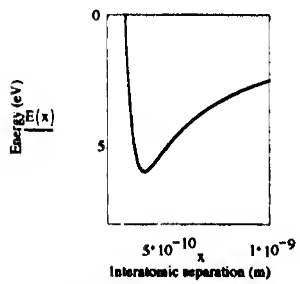
$q := 1.6 \cdot 10^{-19}$ $\quad \epsilon_0 := 8.85 \cdot 10^{-12}$ $\quad M := 1.76267$

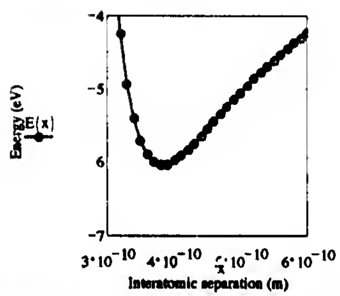
$\rho_0 := 0.034 \cdot 10^{-9}$ $\quad B := 4.512 \cdot 10^{4} \cdot q$

x is the interatomic separation $\quad x := 0.01 \cdot 10^{-9}, 0.02 \cdot 10^{-9} .. 1 \cdot 10^{-9}$

$$E(x) := \frac{\dfrac{-M \cdot q^2}{4 \cdot \pi \cdot \epsilon_0 \cdot x} + B \cdot e^{\frac{-x}{\rho_0}}}{q}$$

This is the energy per ion in electron volts.

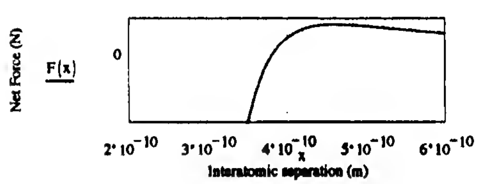




Minimum energy is -6.1 eV so the bonding energy is 6.1 eV. Equilibrium separation is 0.38 nm.

The net force is the derivative of the energy curve. We take E(x) in Joules to obtain F in Newtons.

$$F(x) := \frac{d}{dx}\left[\frac{-M \cdot q^2}{4 \cdot \pi \cdot \epsilon_0 \cdot x} + B \cdot e^{\frac{-x}{\rho_0}}\right] \qquad F(x) := \frac{1}{4} \cdot M \cdot \frac{q^2}{\left[\pi \cdot \left(\epsilon_0 \cdot x^2\right)\right]} - \frac{B}{\rho_0} \cdot \exp\left(\frac{-x}{\rho_0}\right)$$



Declarations

Constants

$q = 1.6022\times10^{-19}$ $\quad c = 299792458$ $\quad \mu_0 = 4\pi \cdot 1\times10^{-7}$ $\quad \epsilon_0 = 8.8542\times10^{-12}$

$h = 6.6261\times10^{-34}$ $\quad \hbar = 1.0546\times10^{-34}$

$k_B = 1.3807\times10^{-23}$ $\quad R_{gas} = 8.3145$ $\quad N_A = 6.0221\times10^{23}$

$g = 9.8066$ $\quad G_{gravitation} = 6.6726\times10^{-11}$

$m_e = 9.1094\times10^{-31}$ $\quad amc = 1.6605\times10^{27}$

$a_0 = 5.2918\times10^{-11}$

$C_{WFL} = 2.443\times10^{-8}$ $\quad C_{Stefan} = 5.6705\times10^{-8}$

(a) Si has the diamond crystal structure.

$a = 0.543 \cdot 10^{-9}$ m

$M_{at} = 28.09 \cdot 10^{-3}$ kg / mole

Si has the diamond crystal structure with 8 atoms in the unit cell.

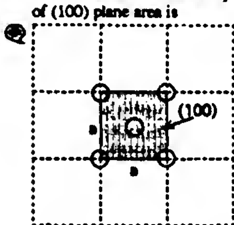
$n = \dfrac{8}{a^3 (10^9)^3}$

$n = 49.968$ Calculate atoms / nm^3

$\rho = \dfrac{8 \dfrac{M_{at}}{N_A}}{a^3}$

$\rho = 2330.7$ Calculate kg m^{-3} or 2.33 g cm^{-3}

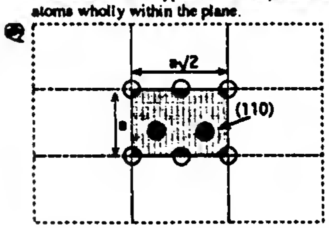
(b) The (100) plane 4 shared atoms at the corners and 1 unshared atom at the center. The corner atom is shared by 4 (100) type planes. Number of atoms per square nm of (100) plane area is

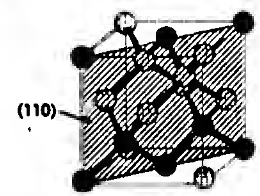


$n_{100} = \dfrac{4\frac{1}{4}+1}{a^2 (10^9)^2}$

$n_{100} = 6.7831$ Calculate atoms per nm^2

The (110) plane is shown below: There are 4 atoms at the corners and shared with neighboring planes (hence each contributing a quarter), 2 atoms on upper and lower sides shared with upper and lower planes (hence each atom contributing 1 / 2) and 2 atoms wholly within the plane.

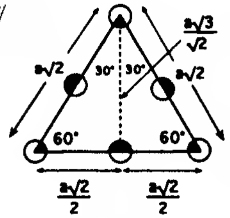


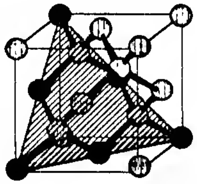


$n_{110} = \dfrac{4\frac{1}{4}+2\frac{1}{2}+2}{\left(a\left[a\sqrt{2}\right]\right)(10^9)^2}$

$n_{110} = 9.5928$ Calculate atoms per nm^2

The (100) plane is shown below:





$$n_{111} = \frac{3\frac{60}{360}+3\frac{1}{2}}{\left(2\frac{1}{2}\left[a\frac{\sqrt{2}}{2}\right]\left[a\frac{\sqrt{3}}{\sqrt{2}}\right]\right)(10^9)^2}$$

$\triangle\ n_{111} = 7.8325$ Calculate atoms per nm^2

(c) Vacancy concentration in Si at 27°C and 1200°C:

$T = 300$ K

Concentration of Si atoms:

$N_{Si} = \frac{8}{a^3}$

$\triangle\ N_{Si} = 4.9968\times10^{28}$ Calculate Si atoms / m^3

$E_v = 2.4q$ Joules / atom

Concentration of vacancies is given by

$$n = N_{Si}\exp\left(-\frac{E_v}{k_B T}\right)$$

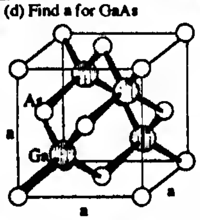
$\triangle\ n = 2.4027\times10^{-12}$ Calculate vacancies / m^3

$T' = 1200 + 273$

$$n' = N_{Si}\exp\left(-\frac{E_v}{k_B T'}\right)$$

$\triangle\ n' = 3.0711\times10^{20}$ Calculate vacancies / m^3

(d) Find a for GaAs



There are 4 Ga atoms and 4 As atoms in the unit cell as shown. Given,

$\rho_{GaAs} = 5.318\cdot10^3$ kg / m^3

$M_{Ga} = 69.72\cdot10^{-3}$ kg/ mole

$M_{As} = 74.92\cdot10^{-3}$ kg/ mole

From the definition of density

$$\rho_{GaAs} = \frac{4\frac{M_{Ga}}{N_A}+4\frac{M_{As}}{N_A}}{a_{GaAs}^3}$$

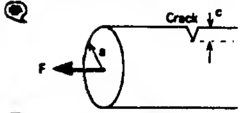
$$\triangle\ a_{GaAs} = \left(\frac{4\frac{M_{As}}{N_A}+4\frac{M_{Ga}}{N_A}}{\rho_{GaAs}}\right)^{\frac{1}{3}}$$ Isolate

$\triangle\ a_{GaAs} = 5.6531\times10^{-10}$ Calculate m or $a = 0.565$ nm

Declarations

Constants

1996 - EE217 - Final - Q.5



Fiber radius is a,

$a = \frac{100}{2}\cdot10^{-6}$ m

Applied tensile load is

$F = 0.1$ N

Glass surface tension is

$\gamma = 1$ J m^{-2}

Elastic modulus is

$E = 70\cdot10^9$ Pa

Radius of curvature is

$r = 0.2\cdot10^{-9}$

$\sigma = \frac{F}{\pi a^2}$ $\triangle\ \sigma = 1.2732\times10^7$ Calculate Pa

$$\sigma_{max} = \sigma\cdot2\sqrt{\frac{c}{r}}$$

$$\triangle\ \sigma_{max} = 2\frac{F\sqrt{\frac{c}{r}}}{\pi a^2}$$ Substitute

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi c}}$$

$\triangle\ \sigma_c = 12.192$ Calculate

Fracture occurs when $\sigma_{max} = \sigma_c$

$\sigma_{max} = \sigma_c$

$$\triangle\ \sigma_{max} = \sqrt{2\frac{\gamma E}{\pi c}}$$ Substitute

$$\triangle\ 2\frac{F\sqrt{\frac{c}{r}}}{\pi a^2} = \sqrt{2\frac{\gamma E}{\pi c}}$$ Substitute

$$\triangle\ 2\frac{F}{\pi a^2} = \sqrt{2\frac{\gamma E}{\pi c}}\frac{1}{\sqrt{\frac{c}{r}}}$$ Move Over

$$\triangle\ 2\frac{F}{\pi a^2} = \frac{\sqrt{2}\sqrt{\gamma}\sqrt{E}\sqrt{r}}{\sqrt{\pi}\,c}$$ Expand

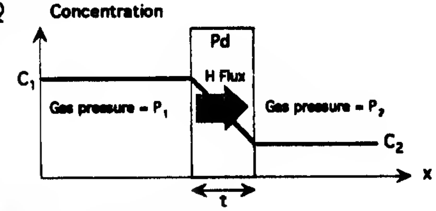
$$\triangle\ c = \frac{1}{2}\frac{\sqrt{2}\sqrt{\gamma}\sqrt{\pi}\sqrt{E}\,a^2\sqrt{r}}{F},$$ Isolate

$\triangle\ c = 1.1724\times10^{-7}$ Calculate m

or $c = 0.117$ microns

Declarations

Constants



Thickness of Pd sheet is

$t = 5\cdot10^{-3}$ m

$C_1 = 2.4$ kg / m^3

$C_2 = 0.6$ kg / m^3

Surface area of Pd

Area $= 0.2$ m^2

$D_0 = 4.5\cdot10^{-7}$ m^2 s^{-1}

$T = 500 + 273$ K

$Q_{diff} = 24.6\cdot10^3$ J / mole

Diffusion coefficient

$$D = D_0 \exp\left(-\frac{Q_{diff}}{R_{gas}T}\right)$$

$D = 9.7935\times10^{-9}$ Calculate m^2 s^{-1}

Apply Fick's Fist Law to find the hydrogen flux, Γ

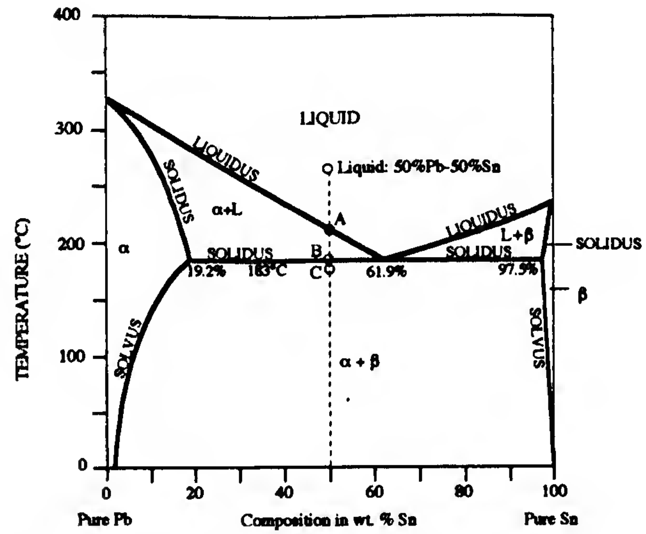
$$\Gamma = D\frac{C_1 - C_2}{t}$$

$\Gamma = 3.5256\times10^{-6}$ Calculate kg m^{-2} s^{-1}

Time = 1·60·60 seconds

Mass = Γ Time Area

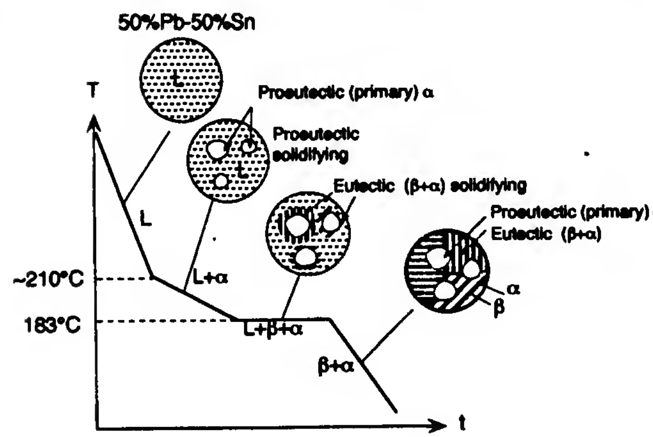
Mass = 0.0025385 Calculate kg



1996-EE217 Final Exam - Q. 6

All compositions are in wt. %

When 50%Pb-50%Sn is cooled from the molten state down to room temperature, it begins to solidify at A at about 210°C. Between 210°C and 183°C, two phases, solid (α) and liquid (L) coexist. At 183 °C, the liquid has the eutectic composition and undergoes the eutectic transformation to become the eutectic solid. Below 183°C, all the liquid has solidified and the structure is the solid α phase and the eutectic structure which is composed of α and β layers.



At 182°C, the composition of the proeutectic or primary α is given by the solubility limit of Sn in α: **19.2 %Sn**.
Primary or proeutectic α (pro-α) exists just above and below 183 °C (eutectic temperature) i.e. it is stable just above and below 183 °C. Thus the mass of pro-α at 182 °C is the same as at 184 °C. Apply the lever rule

$$W_{pro\alpha} = \frac{C_L - C_0}{C_L - C_\alpha}$$
$$W_{pro\alpha} = \frac{61.9-50}{61.9-19.2} = \mathbf{0.28}$$

Mass of primary or proeuectic α is thus **0.28 kg**. The mass of the eutectic is thus 1–0.28 or 0.72 kg.
If we apply the lever rule at 182 °C at 50 %Sn we obtain the weight percentage of α in the *whole* solid:

$$W_\alpha = \frac{C_\beta - C_0}{C_\beta - C_\alpha}$$
$$W_\alpha = \frac{97.5-50}{97.5-19.2} = \mathbf{0.606}$$

Thus the mass of α in the whole solid is 0.606 kg. Of this, 0.28 kg is in the primary (proeuectic) α phase. Thus 0.606–0.28 or **0.326 kg** of α is in the euectic.

$$C_\beta = \frac{50-19.2}{97.5-19.2} \approx 0.393 \quad \text{in the eutectic}$$

Heat capacity of metals is given as 3R per mole or 3k per atom.

Declarations

Constants

1996 - EE217 - Final - Q.6 (e): Calculate the specific heat of the 50%Pb- 50%Sn solder given the atomic masses of Pb, 207.2, and that of Sn, 118.71.

$M_{Pb} = 207.2$ grams / mole

$M_{Sn} = 118.71$ grams / mole

n = number of moles and W_{Pb} = Weight fraction of Pb in alloy

$1 = n_{Pb} + n_{Sn}$

$n_{Sn} = -n_{Pb} + 1$ Isolate

$W_{Pb} = 0.5$

$$W_{Pb} = \frac{n_{Pb} M_{Pb}}{n_{Pb} M_{Pb} + n_{Sn} M_{Sn}}$$

$$W_{Pb} = \frac{M_{Pb} n_{Pb}}{(-n_{Pb}+1) M_{Sn} + M_{Pb} n_{Pb}} \quad \text{Substitute}$$

$$W_{Pb} = \frac{M_{Pb} n_{Pb}}{M_{Sn} + M_{Pb} n_{Pb} - M_{Sn} n_{Pb}} \quad \text{Expand}$$

$(M_{Sn} + M_{Pb} n_{Pb} - M_{Sn} n_{Pb}) W_{Pb} = M_{Pb} n_{Pb}$ Move Over

$M_{Sn} W_{Pb} + M_{Pb} n_{Pb} W_{Pb} - M_{Sn} n_{Pb} W_{Pb} = M_{Pb} n_{Pb}$ Expand

$M_{Pb} n_{Pb} W_{Pb} - M_{Sn} n_{Pb} W_{Pb} = -M_{Sn} W_{Pb} + M_{Pb} n_{Pb}$ Move Over

$M_{Pb} n_{Pb} W_{Pb} - M_{Sn} n_{Pb} W_{Pb} - M_{Pb} n_{Pb} = -M_{Sn} W_{Pb}$ Move Over

$n_{Pb}([M_{Pb} - M_{Sn}] W_{Pb} - M_{Pb}) = -M_{Sn} W_{Pb}$ Collect

$$n_{Pb} = -\frac{M_{Sn} W_{Pb}}{(M_{Pb} - M_{Sn}) W_{Pb} - M_{Pb}} \quad \text{Isolate}$$

$n_{Pb} = 0.36424$ Calculate

$n_{Sn} = 0.63576$ Calculate

Mole fraction of Pb is 0.36 and that of Sn is 1 – 0.364 = 0.636

H capacity of a metal is 3 R per mole. We want specific heat capacity C_s or heat capacity per gram of alloy. 1 mole of the alloy has a mass M_{PbSn}

$M_{PbSn} = n_{Pb} M_{Pb} + n_{Sn} M_{Sn}$

Thus the specific heat capacity (i.e. heat capacity per gram) is,

$$C_s = \frac{3R_{gas}}{n_{Pb} M_{Pb} + n_{Sn} M_{Sn}}$$

$$C_s = \frac{3R_{gas}}{150.94} \quad \text{Calculate}$$

$C_s = 0.16525$ Calculate Joules per K per gram

Declarations

Constants

1996 - EE217 - Final - Q.7

$\mu_{drift} = 56 \cdot 10^{-4}$ $m^2 V^{-1} s^{-1}$

$M_{Ag} = 107.57$ $g\ mol^{-1}$

Density is D

$D = 10.5$ $g\ cm^{-3}$

Number of Ag atoms per m^3

$n_{Ag} = \dfrac{(D \cdot 10^3) N_A}{M_{Ag} \cdot 10^{-3}}$

$n_{Ag} = 5.8783 \times 10^{28}$ Calculate

Assume 1 conduction electron per Ag atom.

$n = n_{Ag}$

$\sigma = n\, q\, \mu_{drift}$

$\sigma = 5.2741 \times 10^{7}$ Calculate $1 / (\Omega\ m)$

Resistivity is $1 / \sigma$

$\dfrac{1}{\sigma}$ $\dfrac{1}{\sigma} = 1.8961 \times 10^{-8}$ Calculate $\Omega\ m$

$\dfrac{\frac{1}{\sigma}}{1.6 \cdot 10^{-8}}$ $\dfrac{\frac{1}{\sigma}}{1.6 \cdot 10^{-8}} = 1.185$ Calculate

$\dfrac{\frac{1}{\sigma} - 1.6 \cdot 10^{-8}}{1.6 \cdot 10^{-8}}$ $\dfrac{\frac{1}{\sigma} - 1.6 \cdot 10^{-8}}{1.6 \cdot 10^{-8}} = 0.18504$ Calculate

The experimental value is 18.5% lower because Ag atoms, on average, may contribute more than 1 electron per atom, i.e. 1.19 electrons per Ag atom.

Given TCR at 0 °C:

$\alpha = \frac{1}{202}$

$To = 0 + 273$

The resistivity obeys

$\rho = K\, T^n$

At $T = To$, $\rho = \rho o$

$\rho o = K\, To^n$

Definition of α is

$\alpha = \dfrac{d\rho}{\rho o\ dT}$

$\alpha = \dfrac{K\, n\, T^{n-1}}{\rho o}$ Substitute

$\alpha = \dfrac{n\, T^{n-1}}{To^n}$ Substitute

At $T = To$ where α is know we have

$T = To$ and

$\alpha = \dfrac{n\, T^{n-1}}{To^n}$

$\alpha = \dfrac{n}{To}$ Substitute

$n = To\, \alpha$ Isolate $n = 1.3515$ Calculate

Declarations

1996 - EE217 - Final - Q.8

Constants

$C_{WFL} = 2.443 \times 10^{-8}$

ρo = Resistivity of Cu matrix; C = Nordheim's coefficient, X = Atomic fraction of Zn in brass

$\rho o = 17 \cdot 10^{-9}$ $C = 300 \cdot 10^{-9}$ $X = 0.2$

$\rho Brass = \rho o + C\, X (1 - X)$

$\rho Brass = 6.5 \times 10^{-8}$ Calculate Ω m or 65 nΩ m

Temperature is

$T = 27 + 273$

Wiedemann - Franz - Lorenz Law is:

$\dfrac{\kappa Brass}{\frac{1}{\rho Brass} T} = C_{WFL}$

$\kappa Brass = \dfrac{C_{WFL}\, T}{\rho Brass}$ Isolate

$\kappa Brass = 112.76$ Calculate $W m^{-1} K^{-1}$

Given the bradd disk dimensions,

$r = \frac{40}{2} \cdot 10^{-3}$ $L = 5 \cdot 10^{-3}$

$\theta Brass = \dfrac{L}{\kappa Brass (\pi r^2)}$

$\theta Brass = 0.035288$ Calculate K / W

Rate of heat flow is Q',

$Q' = 100$ W

Heat flow is due to a temperature difference ΔT,

$Q' = \dfrac{\Delta T}{\theta Brass}$

$\Delta T = Q'\, \theta Brass$ Isolate $\Delta T = 3.5288$ Calculate K

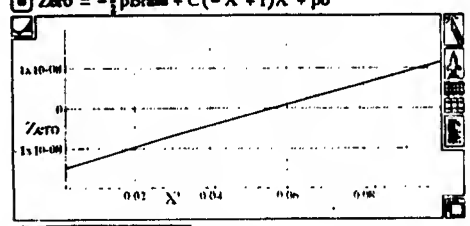
To get half the temperature drop we need half the thermal resistance and hence double the thermal conductivity. That means half the electrical resistance.

Let the new brass composition be X' (atomic fraction of Zn)

$\dfrac{\rho Brass}{2} = \rho o + C\, X'(1 - X')$

$0 = -\frac{1}{2} \rho Brass + C(-X' + 1) X' + \rho o$ Move Over

$Zero = -\frac{1}{2} \rho Brass + C(-X' + 1) X' + \rho o$



$X' = 0.054654$

$Zero = 0$

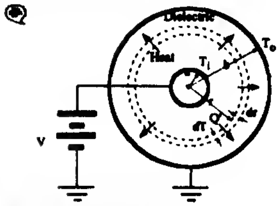
The composition X' = 5.46% Zn.

Declarations

1996 - EE217 - Final - Q.9

Constants

Upon Transform transform $\ln(m) - \ln(w)$ into $\ln\left(\frac{m}{w}\right)$



The joule heat generated in the core conductor flows outwards radially through the dielectric material. The dashed concentric circles identify a thin cylindrical shell concentric with the core.

Given in S.I. units:

$n = 1.15$ $\rho o = 17 \cdot 10^{-9}$ $\kappa = 0.35$

$a = \frac{4}{2} \cdot 10^{-3}$ $b = a + 3 \cdot 10^{-3}$ $b = 0.005$ Calculate

For most metals we can write

$\rho = \rho o\left(\frac{T}{To}\right)^n$

The resistance R of the inner conductor is

$R = \frac{\rho L}{\pi a^2}$

$R = \frac{\rho o\, L\left(\frac{T}{To}\right)^n}{\pi a^2}$ Substitute

$Power = I^2 R$

$Power = \frac{\rho o\, I^2 L\left(\frac{T}{To}\right)^n}{\pi a^2}$ Substitute

Q' = Heat flow

Heat flows through the surface area A of the cylidrical shell of radius r:

$A = 2\pi r L$

$Q' = -(2\pi r L)\kappa \frac{dT}{dr}$

In the steady state, electrical power, Power, generated by the Joule heating of the inner conductor is equal to the the rate of heat flow Q ' through the insulator:

$Power = Q'$

$\frac{\rho o\, I^2 L\left(\frac{T}{To}\right)^n}{\pi a^2} = Q'$ Substitute

$\frac{\rho o\, I^2 L\left(\frac{T}{To}\right)^n}{\pi a^2} = -2\pi\kappa L r (dT)\frac{1}{dr}$ Substitute

$\frac{1}{2}\frac{\rho o\, I^2\left(\frac{T}{To}\right)^n}{\pi^2 \kappa a^2 r} = -(dT)\frac{1}{dr}$ Move Over

$\frac{1}{2}\frac{\rho o\, I^2\left(\frac{T}{To}\right)^n}{\pi^2 \kappa a^2 r} dr = -dT$ Move Over

$\frac{1}{2}\frac{\rho o\, I^2}{\pi^2 \kappa a^2 r} dr = -\frac{1}{\left(\frac{T}{To}\right)^n} dT$ Move Over

$\left(\frac{1}{2}\frac{\rho o\, I^2}{\pi^2 \kappa a^2}\right)\left(\int_a^b \frac{dr}{r}\right) = \int_{Ti}^{To}\left(-\frac{1}{\left[\frac{T}{To}\right]^n}\right) dT$

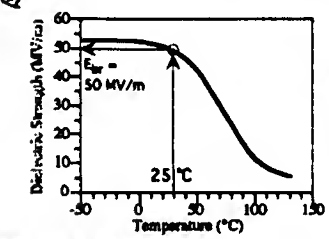
$\frac{1}{2}\frac{(-\ln[a] + \ln[b])\rho o\, I^2}{\pi^2 \kappa a^2} = -\frac{To}{-n+1} + \frac{\left(\frac{Ti}{To}\right)^{-n+1} To}{-n+1}$ Simplify

$\frac{1}{2}\frac{\ln\left(\frac{b}{a}\right)\rho o\, I^2}{\pi^2 \kappa a^2} = -\frac{To}{-n+1} + \frac{\left(\frac{Ti}{To}\right)^{-n+1} To}{-n+1}$ Transform

$\frac{Ti^{-n+1}}{To^{-n+1}} = \frac{1}{2}\frac{(-n+1)\rho o\, I^2 \ln\left(\frac{b}{a}\right)}{\pi^2 \kappa\, To\, a^2} + 1$ Simplify

The above can be written as:

$\frac{To^{n-1}}{Ti^{n-1}} = 1 - \frac{1}{2}\frac{(n-1)\rho o\, I^2 \ln\left(\frac{b}{a}\right)}{\pi^2 \kappa\, To\, a^2}$ Equation(1)



Typical temperature dependence of the dielectric strenth of a polyethylene based polymeric insulation.

At room temperature, 25 °C,

$Ebr = 50 \cdot 10^6$

The maximum field must be 3 times LESS than this breakdown field,

$Emax = \frac{1}{3} Ebr$

$Emax = 1.6667 \times 10^7$ Calculate

But the maximum field depends on the voltage V via

$Emax = \frac{V}{a \ln\left(\frac{b}{a}\right)}$

$V = a \ln\left(\frac{b}{a}\right) Emax$ Isolate

$V = 30543$ Calculate

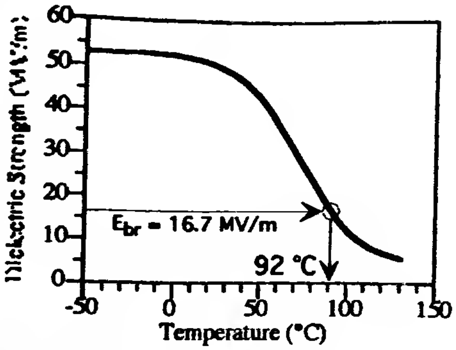
or 13.5 kV, without any current I. NOTE: I = 0 because we assumed Ti = 25 °C, or the inner conductor does not heat up which is only possible if there is no current. In Eqn. (1), Ti = To only when I = 0.

Now, the applied voltage is kept the same (V = 13.5 kV) and the cable carries a current I.

NOTE 1: Since V = 13.5 kV = constant, Emax remains the same 16.7 MV/ m. Because Emax depends on V and cable dimensions only.

NOTE 2: There is now a current I. The inner conductor therefore heats up and its temperature rises along Eqn (1). AsTi increases from 25 °C, Ebr decreases. As I increases, Ti decreases even more. Eventually Ebr decreases so much that it reaches Emax. This happens when Ti ≈ 92 °C. We can find the current from Eqn. (1)

2)



Breakdown occurs when Ti $= 92 + 273$ K.

$Ti = 92+273$

The outer temperature is given as the ambient temperature; say 25 °C

$To = 25+273$

FromEqn(1)

$$\frac{To^{\,n-1}}{Ti^{\,n-1}} = 1 - \frac{1}{2}\,\frac{(n-1)\rho o\, I^2 \ln\left(\frac{b}{a}\right)}{\pi^2 \kappa\, To\, a^2}$$ Equation (1)

$$I = \sqrt{2\,\frac{\left(-\frac{To^{\,n-1}}{Ti^{\,n-1}}+1\right)\pi^2\kappa\, To\, a^2}{(n-1)\rho o \ln\left(\frac{b}{a}\right)}}$$ Isolate

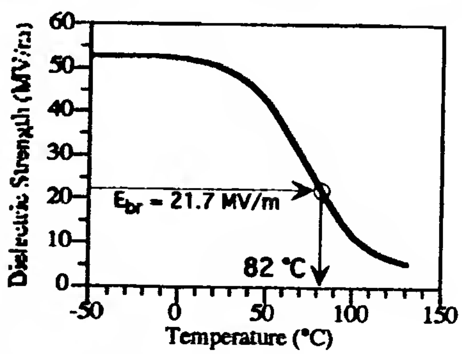
$I = 324.97$ Calculate Amps

When V is 30% larger so is Emax.. New Emax is

$Emax' = 1.3\, Emax$

$Emax' = 2.1667\times10^{7}$ Calculate

The new temperature Ti' when Ebr reaches Emax' is 82 °C as shown below



$Ti' = 85+273$

the new current is I'

$$\frac{To^{\,n-1}}{Ti'^{\,n-1}} = 1 - \frac{1}{2}\,\frac{(n-1)\rho o\, I'^2 \ln\left(\frac{b}{a}\right)}{\pi^2 \kappa\, To\, a^2}$$

$$I' = \sqrt{2\,\frac{\left(-\frac{To^{\,n-1}}{Ti'^{\,n-1}}+1\right)\pi^2\kappa\, To\, a^2}{(n-1)\rho o \ln\left(\frac{b}{a}\right)}}$$ Isolate

$I' = 309.29$ Calculate Amps

Marks

10 2. The time it takes a cell to divide is normally distributed with an average time of one hour and a standard deviation of 5 minutes.
   (a) What is the probability that a cell divides in less than 45 minutes?
   (b) What is the probability that it takes a cell more than 65 minutes to devide?
   (c) What is the time that it takes approximately 98% of all cells to complete their division?

Ans:

(a)

$$z_1 = \frac{45-60}{5} = -3$$

P_r**(a cell divides in LT 45 mins)** $= \Phi(-3) = \mathbf{0.0013}$

(b)

$$z_2 = \frac{65-60}{5} = 1$$

P_r**(a cell divides in MT 65 mins)** $= 1 - \Phi(1) = 1 - \mathbf{0.8413} = \mathbf{0.1587}$

(c)

From the Normal Table: $\Phi(2) \approx 0.98$

Therefore $z_3 = 2$. Find the corresponding x.

$$2 = \frac{x-60}{5}$$

$x = 70$ **mins**

It takes 70 mins for 98% of all cells to complete their division.

University of Saskatchewan
Department of Electrical Engineering
EE 216.3 Probability, Statistics and Numerical Methods
Midterm Examination 2002

Marks

10 1. Cooking oil is produced in two main varieties: mono- and polyunsaturated. Two common sources of cooking oil are corn and canola. The following table shows the number of bottles of these oils at a supermarket:

		Type of Oil	
		Canola	Corn
Type of unsaturation	mono	8	13
	poly	92	77

If two bottles of oil are selected at random without replacement:
   (a) What is the probability that one is monounsaturated canola and the other is polyunsaturated corn oil?
   (b) What is the probability that both bottles are polyunsaturated canola oil?
   (c) What is the probability that the second bottle is monounsaturated canola oil given that the first one was polyunsaturated corn oil?

Ans:

The bottles of oils in the supermarket as mentioned in the Table are:

8 bottles of monounsaturated canola
13 bottles of monounsaturated corn
92 bottles of polyunsaturated canola
77 bottles of polyunsaturated corn

There are total 190 bottles of cooking oil in the supermarket.

(a) There are two ways that a monounsaturated canola and a polyunsaturated corn oil can be picked up; the first bottle could be a monounsaturated canola and the second bottle could be a polyunsaturated corn or, the first bottle could be a polyunsaturated corn and the second bottle could be a monounsaturated canola.

$$P_r = \frac{8}{190} \times \frac{77}{189} + \frac{77}{190} \times \frac{8}{189} = 0.034$$

(b) P_r**(both bottles are polyunsaturated canola)** $= \frac{92}{190} \times \frac{91}{189} = 0.233$

(c) The first selection left 189 bottles to choose from.

P_r**(second bottle is monounsaturated canola given that the first one was polyunsaturated corn)** $= \frac{8}{189} = 0.042$

Marks

10 4. A book club announces a sweepstakes in order to attract new subscribers. The prizes and the corresponding chances are listed below:

Prize	Chance
$100,000	1 in one million
$5,000	1 in 250,000
$100	1 in 5,000
$50	1 in 500

Suppose you have just mailed in a sweepstakes ticket. (a) Calculate your expected winnings. (b) What is the probability that you may win $100 or less?

Ans:

(a)

Expected winnings:

$$E(\textbf{winnings}) = \$100{,}000\times(1/10^6)+\$5{,}000\times(1/250{,}000)+\$100\times(1/5{,}000)+\$50\times(1/500)$$
$$=\$0.24$$

(b)

$$P_r(\text{win } \$100 \text{ or less}) = P_r(\text{win } \$100)+P_r(\text{win } \$50)+P_r(\text{win } \$0)$$
$$=1-P_r(\text{win } \$100{,}000)-P_r(\text{win } \$5{,}000)$$
$$=1-(1/10^6)-(1/250{,}000)$$
$$=0.99999875$$

Marks

10 3. A population consists of one of each of the four numbers 4, 7, 11, 13.
(a) Find the mean and standard deviation of the population.
(b) Consider all possible samples of size two which can be drawn without replacement from this population. Find the mean of each possible sample and use this information to calculate the mean of the sample means and the standard error of the mean.

Ans:

(a)

Mean,

$$\mu=\frac{4+7+11+13}{4}=8.75$$

Variance,

$$\sigma^2=\frac{\sum_i (x_i-\mu)^2}{N}=\frac{(4-8.75)^2+(7-8.75)^2+(11-8.75)^2+(13-8.75)^2}{4}$$
$$=\frac{48.75}{4}=12.188$$

St. deviation, $\sigma = 3.491$

(b)

Possible samples of size two and their means are shown in the following table:

Sample	4, 7	4, 11	4, 13	7, 11	7, 13	11, 13
Mean, $\bar{x}_i$	5.5	7.5	8.5	9	10	12

Mean of the sample means, $\mu_{\bar{x}}=\dfrac{5.5+7.5+8.5+9+10+12}{6}=8.75$

Variance of the sample means, $\sigma_{\bar{x}}^2=\dfrac{\sum_i(\bar{x}_i-\mu_{\bar{x}})^2}{6}$

$$\sigma_{\bar{x}}^2=\frac{(5.5-8.75)^2+(7.5-8.75)^2+(8.5-8.75)^2+(9-8.75)^2+(10-8.75)^2+(12-8.75)^2}{6}=4.0625$$

Standard error of the mean, $\sigma_{\bar{x}}=2.0156$

10 5. Suppose 15% of the trees in a forest have severe leaf damage from air pollution. If 5 trees are selected at random, find the probability that

(a) Three of the selected trees have severe leaf damage.
(b) No more than two have severe leaf damage.
(c) None of them have severe leaf damage.

Ans:

$P_r(\textbf{severe leaf damage}) = 0.15$

$P_r(\textbf{no severe leaf damage}) = 1 - 0.15 = 0.85$

(a) $P_r(\textbf{3 with severe leaf damage}) = {}_5C_3(0.15)^3(0.85)^2 = 0.0244$

(b) $P_r(\textbf{no more than 2 have severe leaf damage}) = P_0 + P_1 + P_2$

$$= (0.85)^5 + {}_5C_1(0.15)(0.85)^4 + {}_5C_2(0.15)^2(0.85)^3$$
$$= 0.4437 + 0.3915 + 0.1382$$
$$= 0.9734$$

(c) $P_0 = (0.85)^5 = 0.4437$

THE END

24

University of Saskatchewan
Department of Electrical Engineering
EE 216.3 Probability, Statistics and Numerical Methods
Midterm Examination

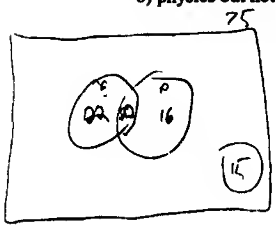
Instructor: N. Chowdhury October 25, 2001

Student Name:

Student ID:

Marks

6 1. In a group of 75 students, 15 take neither English nor physics, 44 take English and 38 take physics. What is the probability that a student chosen at random from this group takes:
a) both English and physics?
b) physics but not English?



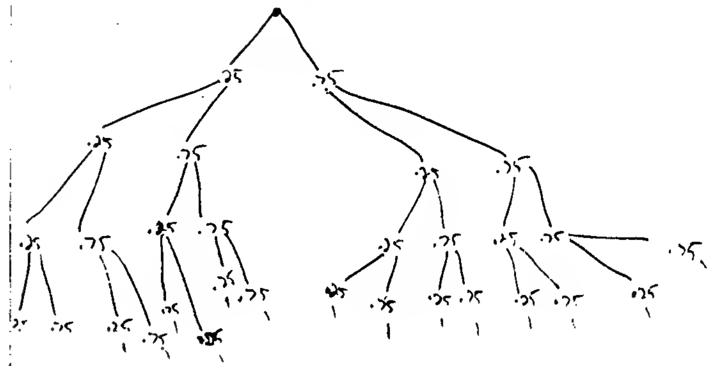
a) $\frac{22}{75} = .2933$ ✓

B) $\frac{16}{75} = .2133$ ✓

6

Marks

9 2. The probability that Mark will hit a target on a certain range is 25% for any one shot, regardless of what happened on the previous shot or shots. He fires four shots.
a) What is the probability that Mark will hit the target exactly twice?
b) What is the probability that he will hit the target at least once?
c) Find the expected number of hits on the target.



a) $[(.25)(.25)(.75)(.75)]6$ ✓

$= .2109$

B) $1-((.75)(.75)(.75)(.75))$

$= .6836$ ✓

9

c) $E(x) = \sum_{all\ x_i} (x_i)(P(x_i))$

$= 1(.75)(.75)(.75)(.25) + 0(.75)(.75)(.75)(.75) + 2(.25)(.75)(.25)(.25) + 1(.75)(.75)(.75)($

$+ 2(P_2) + 1(P_1) + 2(P_2) + 3(.75)(.25)(.25)(.25) + 1(P_1) + 2(P_2) + 3(P_3) + 2(P_2) + 2(P_2) + 3(P_3)$

$+ 3(P_3) + 4(P_4)$

$= 4(1P_1) + 6(2(P_2)) + 4(3(P_3)) + 1(4P_4) + 1(0(P_0))$

$= 1$

Marks

9   3. Twelve doughnuts sampled from a manufacturing process are weighed each day. The probability that a doughnut weighing less than the design weight is 0.18.
a) What is the probability that a sample of twelve doughnuts contains exactly three doughnuts weighing less than the design weight?
b) What is the probability that the sample contains more than three doughnuts weighing less than the design weight?
c) In a sample of twelve doughnuts, what is the expected number of doughnuts weighing less than the design weight? Pr Failure

$-N = 12$

a) $(.82)^{12}(.18)^0 + (.82)^{11}(.18)^1 + (.82)^{10}(.18)^2 + (.82)^9(.18)^3$

$= .1159$ ✗

a) 21.5%

b) $1 - .1159 = .8841$ ✗

B) $P_r[X \geq 4] = 1 - P_{r(1)} - P_{r2} - P_{r3}$ $-P_{r0}$

c) $E(x) = \sum (x_i)\, P(x_i)$

$= .3216$ ✗

c) $(n, P_r\ \text{Failure})$

$(12, (.18))$

0

$std = \sqrt{12\,(.82)(.18)}$

$Var = npq$

Marks

9   4. 2500 electric lamps are installed in a city for street lighting. The burning lifetime of these lamps are normally distributed and have a mean burning life of 1400 hours with a standard deviation of 250 hours.
a) What is the probability that a lamp will fail in the first 800 burning hours?
b) What is the probability that a lamp will fail between 1200 and 1700 burning hours?
c) How many lamps are expected to fail between 1200 and 1700 burning hours?

$\mu = 1400$

$\sigma = 250$

a) $Z_1 = \dfrac{X_1 - \mu}{\sigma} = \dfrac{800 - 1400}{250} = -2.4 \Rightarrow \Phi(-2.4) = 0.0082$

$\therefore P_r[X \leq 800] = 0.0082$ ✓

B) $Z_1 = \dfrac{X_1 - \mu}{\sigma} = \dfrac{1200 - 1400}{250} = -.8 \Rightarrow \Phi(-.8) = 0.2119$

$Z_2 = \dfrac{X_2 - \mu}{\sigma} = \dfrac{1700 - 1400}{250} = 1.2 \Rightarrow \Phi(1.2) = 0.8849$

$\therefore P_r[Z_1 \leq X \leq Z_2] = \Phi(Z_2) - \Phi(Z_1) = 0.8849 - 0.2119 = .673$ ✓

c) $(2500)(.673) = 1682.5$ lamps will fail from 1200 - 1700 hours. ✓

9

THE END

Midterm

University of Saskatchewan, Electrical Engineering
EE 216.3 Probability, Statistics and Numerical Methods
February 15, 2001 K. Takaya

Note: 1.5 hour open book exam. All questions are of equal value.

1. A florist made 20 flower pots of mixed seeds, which were numbered from 1 to 20. The sample space is thus $S = \{1, 2, 3, \cdots, 20\}$. After all flowers have bloomed, one or more red flowers are found in those pots grouped as $R = \{1, 4, 5, 6, 7\}$. One or more yellow flowers are found in $Y = \{2, 6, 7, 8, 10\}$. One or more blue flowers are found in $B = \{3, 4, 7, 9, 10\}$. Those pots that contain none of red, none of yellow, and none of blue flowers are group as others, i.e. $O = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$. Assuming the probability of picking a pot is equally likely, having a probability of $\frac{1}{20} = 0.05$, Answer the following questions.

   (a) Draw the Venn Diagram by clearly showing the events, S, R, Y, B, and O.

   (b) What is the probability $Pr(S)$ of the entire sample space S?

   (c) What is the probability $Pr(R \cap B)$?

   (d) What is the probability $Pr(R \cup B)$?

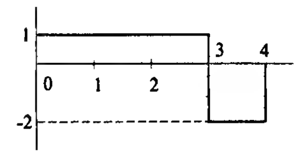
   (e) What is the probability $Pr(\bar{R})$, that is the probability of having a pot that does not contain any red flowers. $\bar{R}$ means NOT red.

   (f) Write a logical expression for having only red flowers.

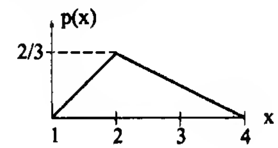
2. There are 10 red Honda Civics on a dealer's lot. The total number of red cars in the lot is 20. The lot contains 60 cars. There are 20 non-red Honda Civics. Let R = a car is red, and H = a car is a Honda Civic. Find $Pr(H|R)$, i.e. the conditional probability that a car is a Honda Civic, given the color is red. Not only the numerical answer, you must write the Bayes's formula applied and used in your calculations.

3. The rhino charging at you will require 3 hits from your 0.22 caliber pistol before it will be sufficiently discouraged to go elsewhere. Under the circumstance, your shooting ability will yield a 0.5 probability of hit. Assuming you will stop shooting after you have achieved 3 hits, what is the probability that exactly 6 shots will be required.

4. Obtain the DC component and rms value of the periodic wave repeating the illustrated waveform shown below with a time interval of 4.



5. The probability density function for a continuous random variable x is given as shown below. The random variable is defined in the range of $1 \le x \le 4$. Calculate the mean $\mu = E\{x\}$ and the variance $\sigma^2 = E\{(x - \mu)^2\}$.



The END.

UNIVERSITY OF SASKATCHEWAN
College of Engineering

Probability and Statistics
GE 210.3

Midterm Exam **October 1999**

Time: 2 hours

All questions have equal marks. Students are allowed to bring in 3 sheets of formulae, etc., but no solved problems.

1) For a large class of about 100 students, the average number of students missing any lecture is 6.4. There are 3 lectures per week in the course. Assume that non-attendance is a random event.
   (a) What is the probability that Monday's lecture has more than 3 students missing?
   (b) What is the probability that there will be fewer than 4 students missing on Monday and more than 4 students missing on Wednesday?
   (c) What is the probability that there will be fewer than 3 students missing for at least 2 of the three lectures for the week?
   (d) What is the probability that the total number of absences for the week is 20?

2) A construction company, prior to designing and erecting a building, tests for foundation conditions. It is known that they will find lake deposits, glacial till, or sand with a probability of 0.7, 0.2, and 0.1, respectively. It is also known that, although the design will be optimized for conditions found, there will be costs during and after construction caused by two factors. If the foundation material is lake deposit, there is a 0.3 probability of differential settlement causing damage of \$250,000 or a 0.05 probability of seepage causing damage of \$150,000. If the material is glacial till, there is a 0.05 probability of differential settlement causing damage of \$100,000 or a 0.1 probability of seepage causing damage of \$50,000. If the material is sand, there is a 0.15 probability of differential settlement causing damage of \$150,000 or a 0.35 probability of seepage causing damage of \$300,000.

   a) Calculate the expected cost of the damage.
   b) Calculate the standard deviation of the cost of the damage
   c) Calculate the expected cost of the damage if the foundation material is till.
   d) Calculate the probability that, if the cost of the damage for a building constructed in the same area was more than \$100,000, it was due to seepage.

3) A trucker has found that there is a 15% probability that, on any one day, he will have insufficient business to keep the truck busy the entire day.

   a) Calculate the probability that, in a 10 day period, there will be more than one day with insufficient business.

   b) The trucker joins 9 other truckers to form an association. All the truckers experience the same 15% probability of insufficient business on any one day. What is the probability that, for a ten day period, three of the 10 truckers will experience 2 days of insufficient business, four will experience 1 day of insufficient business, and the remainder will experience no days of insufficient business?

4) (a) A probability distribution function has the form:

$$f(x) = \begin{cases} 0 & x<0 \\ bx+cx^2 & 0\le x\le 2 \\ 0 & x>2 \end{cases}$$

   with the mean at x=1.2.
   Determine the constants "b" and "c".

   (b) A probability distribution function has the form:

$$f(x) = \begin{cases} 0 & x<0 \\ x+sx^2 & 0\le x\le 2 \\ 0 & x>2 \end{cases}$$

   What is the probability of x being less than 1.00?

   (c) A probability distribution function has the form:

$$f(x) = \begin{cases} 0 & x<0 \\ gx+hx^2 & 0\le x\le 2 \\ 0 & x>2 \end{cases}$$

   with the mode at x=1.2.
   Determine the constants "g" and "h". (Hint: At the mode, the slope is 0.)

5) At a certain golf club, 3 of the members have been known to cheat on their scores. Golfer A cheats on his score 30% of the time, Golfer B cheats on his score 50% of the time, and Golfer C cheats on his score 70% of the time. Assume that these golfers cheat independently of one another. The three go out together for a game.

   a) Find the probability that at least one of them will cheat.
   b) Find the probability that no more than one will cheat.
   c) ~~If two~~ cheat in the game, what is the probability that Golfer A was the one that didn't cheat?
   d) You join the three for a round of golf; what is the probability that you cheat?

THE END

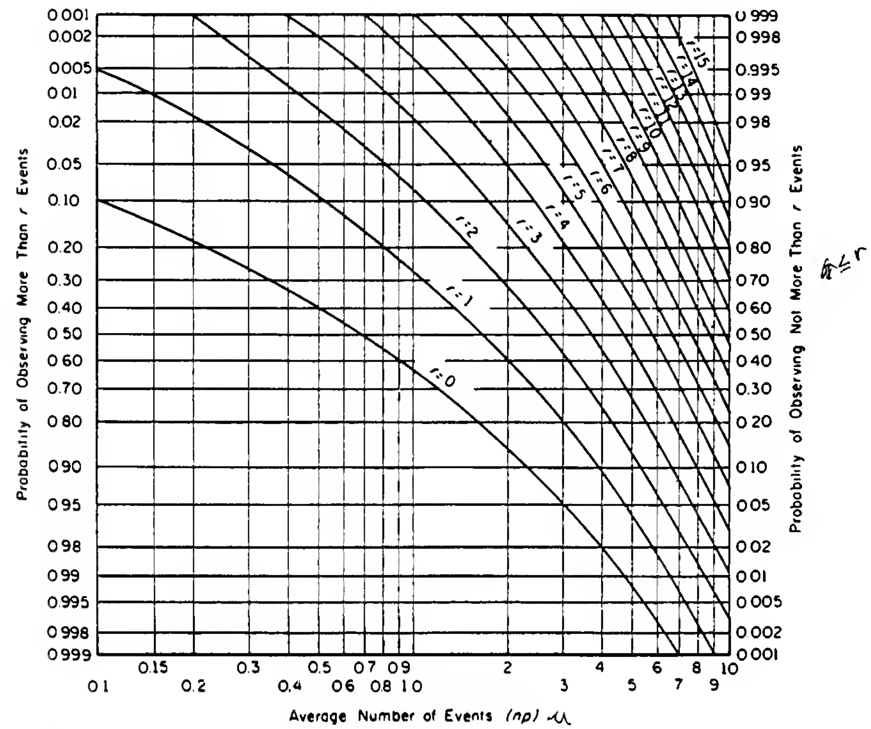


Figure 9.1 Cumulative Poisson distribution.

Kennedy and Neville, 1986, Basic Statistical Methods for Engineers and Scientists

UNIVERSITY OF SASKATCHEWAN
College of Engineering

GE 210.3
Probability and Statistics

Final Exam

Time: 3 hrs. **Date: December 16, 1999**

Note: A student may bring in three sheets (8.5 x 11) of formulae with no solved problems allowed.

MARKS

(16) 1. A large government department tested out three different brands, A, B, & C, of the same type of software, allowing the employees to use the brands according to their preferences. The firm intended to keep the two most used software packages. It was found that of the 60 persons in the department,

- 20 used only one brand,
- 20 used only 2 brands,
- 10 used all 3 brands,
- 30 used brand A
- 15 used both brands A and B
- 8 used only brand C
- 6 used both brands B and C but not A
- some may not have used any

(a) Which brand should the department delete from their selection? B
(b) What is the probability that a person selected at random used packages A and B? 1/4
(c) Given that a person used packages A or B, what is the probability that she also used package C? 2/3

(16) 2. It is known that soil density varies as depth below the surface. At a certain location, soil density (y in Mg/m^3) was measured at 15 depths (x in m). An analysis gave the following results:
$\Sigma x = 5.55$, $\Sigma x^2 = 2.255$, $\Sigma y = 33.15$, $\Sigma y^2 = 73.643$, $\Sigma xy = 12.493$.

(a) Find the regression equation of y on x. $y = 1.79 + 1.13x$
(b) Find the 95% confidence limits of the actual value of y for a value of x=0.42. (i.e. if many samples are taken at a depth x = 0.42, what are the confidence limits for the mean density of these samples.) 1.79 , 2.73
(c) Find the correlation coefficient, r, and test whether it is significantly different from 0. at 5% lofs $r = 0.82$
yes signdiff

(17) 3. A manufacturing plant produces bags of dry concrete mix. The machines filling the bags are set to add to each bag (which itself has a mass of 0.8 kg): 35 kg of sand, 5 kg of cement, and 0.54 kg of additive. The standard deviations of the normally distributed variables are 0.2 kg for the bag, 1.2 kg for the sand, 0.25 kg for the cement, and 0.12 kg for the additive.

(a) How many filled bags must be weighed to be 95% confident that the mean bag mass so determined is within +/- 0.16kg of the true mean bag mass? 471

(b) What are the 99% confidence limits for the mass of a pallet load of 20 filled bags, the mass of the pallet itself being 8.7 kg with a standard deviation of 1.6 kg? 840.65
830.35

(17) 4. A particular type of event is thought to have a constant probability of success of 0.4 for an individual event. It was decided to test this hypothesis by running 150 tests with 6 trials, or events, (n = 6) in each test.

The following results were obtained.

Number of successes per test	# of tests
0	5
1	37
2	39
3	30
4	30
5	7
6	2

Using a 5% level of significance, can it be concluded that the results follow the applicable distribution with p = 0.4? $\chi^2_{calc} = 15.67$ $crit = 14.45$ does not follow

(17) 5. Ace Explosive Demolition is concerned with consistent and reliable burn times for the fuses that they use in their work. They have 2 suppliers of fuses. Tests were run on samples from both suppliers to determine burn times. The results were as follows:

Fuses from Supplier A	Fuses from Supplier B
$n_1 = 30$	$n_2 = 32$
mean = 30.62	mean = 31.37
s = 0.62	s = 0.43

(a) In previous tests of fuses from these 2 suppliers, the variance for burn times of fuses from Supplier A has been significantly higher than that for Supplier B. Using a 5% level of significance, does the current data indicate that this situation is still the same? still higher $F_{calc} = 2.08$ $crit = 1.9$

(b) Assuming that there is no significant difference in the variances, determine, using a 5% level of significance, if there is a significant difference in the mean burn times for the fuses from the 2 suppliers. $t_{calc} = 7.59$ $crit = 2.00$ yes sign diff in means

(17) 6. A survey of customers in an auto repair shop with more than one mechanic showed that 20% had complaints about the service. 50% of the complaints were about mechanic A. Mechanic A performs 40% of the jobs in the shop.

(a) If a job is done by Mechanic A, what is the probability that there will be a complaint? 0.25

(b) If, for a particular job, there is no complaint, what is the probability that the job was not done by Mechanic A? 0.375

(c) The workload in the shop is high, so the manager of the shop hires another mechanic. This reduces the workload by 25% for each of the previous employees. The manager finds that he now has complaints about 30% of the jobs. If the new mechanic carries out a repair on your car, what is the probability that it will be done satisfactorily? Assume that that the performance of the previous employees hasn't changed, i.e. the probability of complaint about service by each mechanic has not changed. 0.4

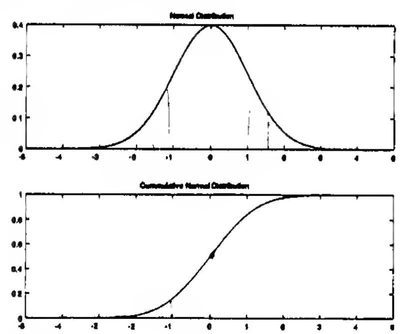
****THE END****

University of Saskatchewan, Electrical Engineering
EE 216.3 Probability, Statistics and Numerical Methods
Final Examination, April 19, 2001 K. Takaya

Note: 3 hour open book exam. Answer all 9 questions. Marks are as indicated in each question.

1. (10) Box I contains 4 white and 1 black ball. Box II contains 2 white and 3 black balls. A box is selected at random, and 3 balls are drawn from it. Given that two balls are white and one is black, what is the probability that Box I was chosen?

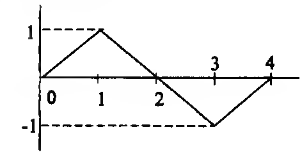
2. (12) The probability density function and cumulative probabilities of the standard Normal distribution are shown in the figure below. The numerical table is also attached at the end of the problem sheets. Answer the following questions.
   (a) When a random variable z obeys the standard Normal distribution, what is the probability that $0 \le z$?
   (b) When a random variable z obeys the standard Normal distribution, what is the probability that $-1 \le z \le 1$?
   (c) When a random variable z obeys the standard Normal distribution, what is the probability that $-1.5 \le z \le -0.5$?
   (d) If the height of the adult population of Canada is a Normal distribution of the mean $\mu = 167.5$ cm and the standard deviation of $\sigma = 7.5$cm, what percentage of Canadian adults are taller than 170cm?



3. (10) A factory producing nails knows that the length of two inch nails has a standard deviation of 0.1 inch.
   (a) Find the probability that the length of a single nail is between 1.9 and 2.1 inches.

   (b) Find the probability of the average length of 100 two inch nails to fall between 1.9 and 2.1 inches.

4. (10) Obtain the rms value of the periodic wave repeating the illustrated waveform shown below with a time interval of 4.



5. (12) The following table shows the results from two separate surveys conducted for 100 persons. One survey (Survey 1) requests a integer value ranging from 1 to 5. The other (Survey 2) expects a numerical answer chosen out of 1, 2 and 3.

Survey 2	Survey 1				
	1	2	3	4	5
1	5	15	10	5	0
2	0	10	20	5	0
3	0	0	5	10	15

   (a) Calculate the mean μ_1 of Survey 1, and the mean μ_2 of Survey 2.
   (b) Calculate the variance σ_1^2 of Survey 1, and the variance σ_2^2 of Survey 2.
   (c) Calculate the covariance σ_{12}^2 between Survey 1 and Survey 2, and the correlation coefficient r.

6. (12) A Matlab program of the Gauss Jordan method that solves a set of 3 simultaneous linear equations for three unknowns

$$\begin{aligned} 3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 &= -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 &= 71.4. \end{aligned}$$

is given as follows. In this Matlab program, every step of Gauss Jordan's elimination method is programmed in one statement. Each row of the matrix M is treated as a unit of operations by `M(1,:)`, `M(2,:)`, `M(3,:)`.

```
M=[3, -0.1, -0.2, 7.85;
   0.1, 7, -0.3, -19.3;
   0.3, -0.2, 10, 71.4]
```

```
M(1,:)=M(1,:)/M(1,1)             % Make M(1,1)=1

M(2,:)=M(2,:)-M(2,1)*M(1,:)      % Make M(2,1)=0
M(3,:)=M(3,:)-M(3,1)*M(1,:)      % Make M(3,1)=0

M(2,:)=M(2,:)/M(2,2)             % Make M(2,2)=1

M(1,:)=M(1,:)-M(1,2)*M(2,:)      % Make M(1,2)=0
M(3,:)=M(3,:)-M(3,2)*M(2,:)      % Make M(3,2)=0

M(3,:)=M(3,:)/M(3,3)             % Make M(3,3)=1

M(1,:)=M(1,:)-M(1,3)*M(3,:)      % Make M(1,3)=0
M(2,:)=M(2,:)-M(2,3)*M(3,:)      % Make M(2,3)=0
```

row 1.

This program can be considerably simplified with the use of double for loops. The simplified program has a structure as shown below. Where, the variable m is row number, and the variable n is column number. Two statements marked as statement 1 and statement 2 are hidden. Write appropriate Matlab statements that should replace these commented hidden statements.

```
for m=1:3
% statement 1.
   for n=1:3
      if n~=m
         % statement 2.
      end
   end
end
```

7. (10) The Fourier series is defined for a periodic function $f(t)$ having one cycle period of T_0 and $\omega_0 = \frac{2\pi}{T_0}$.

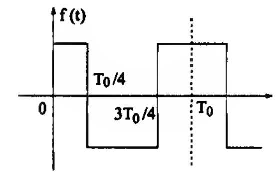
$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$a_0 = \frac{1}{T_0}\int_{T_0} f(t)\,dt$$

$$a_n = \frac{2}{T_0}\int_{T_0} f(t)\cos n\omega_0 t\,dt, \quad b_n = \frac{2}{T_0}\int_{T_0} f(t)\sin n\omega_0 t\,dt$$

Obtain the Fourier series coefficient a_n of a square wave defined by

$$f(t) = \begin{cases} 1 & \text{if } 0 \le t \le \frac{T_0}{4} \\ -1 & \text{if } \frac{T_0}{4} \le t \le \frac{3T_0}{4} \\ 1 & \text{if } \frac{3T_0}{4} \le t \le T_0 \end{cases}$$



8. (12) Manually calculate the four coefficients, b_0, b_1, b_2 and b_3, of the cubic spline function

$$f(t) = b_0 + b_1(t-t_0) + b_2(t-t_0)(t-t_1) + b_3(t-t_0)(t-t_1)(t-t_2)$$

which passes a set of points $(x, y) = (0, 5), (1, 3), (2, 2)$ and $(3, 4)$. Write a complete divided difference table necessary to calculate b_0, b_1, b_2 and b_3.

i	t_i	$f(t_i)$	first	second	third
		$b_0 =$	$b_1 =$	$b_2 =$	$b_3 =$
0	t_0	$f(t_0)$	$f[t_1, t_0]$	$f[t_2, t_1, t_0]$	$f[t_3, t_2, t_1, t_0]$
1	t_1	$f(t_1)$	$f[t_2, t_1]$	$f[t_3, t_2, t_1]$	
2	t_2	$f(t_2)$	$f[t_3, t_2]$		
3	t_3	$f(t_3)$			

9. (12) Draw a simulation block diagram for Simulink that solves a differential equation,

$$2\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + x = f(t).$$

Assume the input function $f(t)$ uses a pulse generator. The components necessary to complete the simulation block diagrams are;

2 integrators
1 sum (adder)
2 or more gains
1 pulse generator
1 scope

II. Multiple Choice Questions (2 marks each): TO BE ANSWERED ON THE OPTIC SCAN SHEET.

**** SELECT THE SINGLE BEST CHOICE FOR EACH OF THE FOLLOWING QUESTIONS****

11. Which of the following identifiers is an invalid identifier in c++?
   A) `a_3`
   B) `_3a`
   C) `3a_`
   D) `cOuNt`
   E) `x`

12. What is the value of variable **Z** after the following code segment has executed?

```
int A = 6,Z = 0;
if (A  >= 5)
  if (A > 10)
    Z = A;
  else
    Z = 2 * A;
```

   A) 0
   B) 6
   C) 12
   D) undefined
   E) none of the above

13. What value does the expression `(1 == 3*2-5)*2` return?
   A) 0
   B) 1
   C) 2
   D) -10
   E) none of the above

14. What is the value of variable `i` after the following code segment has executed?

```
for(int i = 1; i <= 5; i++)
    cout << "*";
```

   A) 0
   B) 1
   C) 5
   D) 6
   E) none of the above

15. Which of the following loop statements will execute the loop-body statements at least once?
   A) `do`
   B) `for`
   C) `while`
   D) both A) and B)
   E) none of the above

Section: 01 (Koehn) / 03 (Zimpel) / 05 (Olorunleke)

Section II = 24

Name:

Student #

Signature

University of Saskatchewan - Department of Computer Science

CMPT 116 Midterm Examination - October 29, 2002

Total Marks: 62 marks **CLOSED BOOK** **Time: 60 Minutes**

Instructions:

PLEASE ANSWER QUESTIONS 1 through 20 ON THE PROVIDED OPTIC SCAN SHEET. Darken with pencil (HB preferred) the circle that corresponds with your answer for each of these questions. Be sure to include your name and student number on the top of the OPTIC SCAN sheet (and on the top of this question sheet).

THE REMAINDER OF THE QUESTIONS (QUESTIONS 21 through 24) ARE TO BE ANSWERED ON THIS QUESTION SHEET. If you don't have enough space, write on the back of the page, indicating clearly that your answer is continued there. Be sure to pace yourself according to the marks allotted to each question. ... good luck!!

I. True / False Questions (1 mark each): TO BE ANSWERED ON THE OPTIC SCAN SHEET.

**** ANSWERS: (A) TRUE (B) FALSE ****

1. `char` is an integer data type in c++. F
2. In c++, a function must return a value. F
3. `byte` is a standard data type in c++. F
4. The expression `(a < b) * (b > a)` will return zero. T
5. A recursive function must not have many base cases. T
6. The expression: `((A*2 >= B) || !(B < 0)) && (X == A)` evaluates to **true**, assuming the following variable declarations have been made: T

```
int A = -5, B = 3;
char X = 'A';
```

7. The order of arguments in a function call is arbitrary. F
8. The c++ compiler detects all errors. F
9. The statement `float z = 1/2` will initialize variable `z` to 0 (zero). F
10. The following code segment will output: **`The answer is yes.`** T

?

```
char answer = 'y';
cout << "The answer is ";
if (answer = 'n')
    cout<< "no." << endl;
else
    cout<< "yes." << endl;
```

5

III. Analyzing & Writing Code: TO BE ANSWERED ON THIS QUESTION SHEET.

21. (6 marks) Trace the following loop.

```
int result = 0, i = 5;
while ( (i > 0) && (result <= i*3) ) {
    result = result + i;
    i--;
}
```

Fuck You

5 i=4 15 12

After the code segment has executed,

a) What are the value of variables:

i) result: 10 X   ii) i: 3 X

b) How many times was the loop body executed (i.e., number of iterations)? 2 X

22. (6 marks) Rewrite the following code segment which contains a series of nested if-then-else statements using one or more **non-nested** if-then-else statements.

```
if (a >= 10)
{
   if (a > 20)
   {
      if (a == 30)
         cout << "is 30";
   }
   else
      cout << "greater than 20";
}
else
   cout << "less than 10";
```

6

```
If(a==30)
  cout<<"is 30";
else if((a>=10) &&(a<=20))
  cout<<"greater than 20";
else if (a<10)
  cout<<"less than 10";
```

2 2 2

16. Which of the following is true?
   A) a function must always be defined before (above) the `main` function
   B) a function must always be defined after (below) the `main` function
   C) a function must always be defined inside the `main` function
   D) a function must always be defined within another function
   E) none of the above statements is correct

17. Which of the following loops will not terminate?
   A) `for(int I=1; I!=10; I=I+2);`
   B) `for(float I=0.3; I<10.0; I=pow(I,2));`
   C) `for(int I=0; I>=10; I=I*I);`
   D) Only loops A) and B) will not terminate
   E) All of the loops A) B) and C) will not terminate

X 18. Suppose that **X** and **Y** have been declared to be variables of type integer and that **fn** is a function with function header:

```
int fn(int &num1, int num2)
```

Which of the following is NOT a valid C++ code fragment?

2

   A) `cout << fn(X, 2*Y);`
   B) `cout << 33.5 / fn(X,Y);`
   C) `Y = fn(X,Y);`
   D) `X = abs(fn(2,Y)) + sqrt(Y);`
   E) `if (fn(X,Y) < 0) {cout << fn(X,Y);}`

19. In C++ source code, programmer comments begin with a:
   A) #
   B) //
   C) $
   D) {
   E) !

X? 20. What does the following function return when the function call **Fn(2,3)** is made?

```
int Fn(int x, int y) {
    if (y == 1)
       return x;
    else
       return x + Fn(x, y-1);
} // end fn Fn.
```

   A) 1
   B) 2
   C) 3
   D) 6
   E) 8

2+2+2

16

-4

24. (10 marks) Write a complete c++ program that calculates and displays all divisors of a positive integer number N. Within your program, you should prompt the user for the integer number N. For example, the divisors of 20 are: 1, 2, 4, 5, 10, 20. (Take abs value of int)

```
#include <math.h>
#include <iostream.h>
int absolute (int); // absolute prototype
void dispDivisors (int); //dispDivisor prototype
Void main ()
{
  int Number =0;
  cout << "Please enter an integer please :";
  cin >> Number;
  dispDivisors (Number);

} //end fn main.

Void dispDivisors (int Num)
{
  int count=0;
  Num = absolute(Num);
  cout << "The divisors of " << Num << " are";
  for (count=0; count <= Num; count++)
  {
     if ((Num % count) == 0)
       cout << Num << ", ";
  } //end for loop.
} //end fn dispDivisors

int absolute (int Num)
{
  if (Num < 0)
    Num = abs(Num);
  return Num;
}
```

9

~ The End ~ */

23. (10 marks)

a) Construct a c++ function named **IsTriangle** that accepts three angles (type float) as parameters and returns the integer value 1 (one) if the angles add up to 180 degrees (error tolerance of the sum less than or equal to 0.001). The function should return the value 0 (zero) if the angles do not add up to 180 degrees. For example, the function call: `IsTriangle(60.001, 59.999, 60.0003)` should return the value 1.

```
int IsTriangle (float Angle1, float Angle2, float Angle3)
{
  float Sum=0;
  Sum=(Angle1 + Angle2 +Angle3);
  if((Sum <= 180.001) &&(Sum >= 179.999))
    return 1;
  else
    return 0;
}
```

5

b) Rewrite the function **IsTriangle** so that the return type of the function is "void" and that the value returned by the function is instead returned through a fourth parameter in the parameter list.

Example function call: `IsTriangle(60.001, 59.999, 60.0003, result)`

```
Void IsTriangle (float Angle1, float Angle2, float Angle3, int& Check)
{
  float Sum =0;
  Sum=(Angle1 +Angle2 +Angle3);
  if ((Sum<=180.001)&&(Sum >= 179.999))
    Check=1;
  else
    Check =0;
  return check;
}
```

4

(17)

Section: 01 (Vassileva) / 03 [illegible] / 05 [illegible]oehn)

Name: ______

Studen ______

Signatu ______

University of Saskatchewan
Department of Computer Science
CMPT 116 Midterm Examination - CLOSED BOOK
Nov 1, 2001

Total Marks: 35 marks **Time: 60 Minutes**

Instructions:

PLEASE ANSWER QUESTIONS 1 through 17 ON THE PROVIDED OPTIC SCAN SHEET. Darken with pencil (HB preferred) the circle that corresponds with your answer for each of these questions. Be sure to include your name and student number on the top of the OPTIC SCAN sheet (and on the top of this question sheet).

THE REMAINDER OF THE QUESTIONS (QUESTIONS 18 through 20) ARE TO BE ANSWERED ON THIS QUESTION SHEET. If you don't have enough space, write on the back of the page, indicating clearly that your answer is continued there. Be sure to pace yourself according to the marks allotted to each question. ... good luck!!

I. True / False Questions (1 mark each): TO BE ANSWERED ON THE OPTIC SCAN SHEET.

1. In C++, `upperLimit2` and `iUpperLimit2` are both valid variable names.

   (A) TRUE (B) FALSE

2. Suppose variable `X=3, Y=1, Z='a'`. Given these values, the following expression evaluates to false. `(Z=='Z') || ((Y<=X-Y) && !(Y==0))`

   false

   (A) TRUE (B) FALSE

3. A program that does ~~not contain~~ any syntax errors will successfully compile.

   ?

   (A) TRUE (B) FALSE

4. Execution of the following C++ code segment will display : `iNum is 10`

```
int iNum = 5;
if (iNum = 10)
    cout << "iNum is 10";
else cout << "iNum is not 10";
```

   (A) TRUE (B) FALSE

5. Integer values can be read via the keyboard or file input into variables of type float.

   (A) TRUE (B) FALSE

6. Using a pass-by-reference parameter, the called function can obtain the initial value of an argument as well as change the value of the argument.

   (A) TRUE (B) FALSE

II. Multiple Choice Questions (1 mark each): TO BE ANSWERED ON THE OPTIC SCAN SHEET.

7. In C++, preprocessor commands begin with a:

   (A) `//`
   (B) `{`
   (C) `#`
   (D) `cout`
   (E) `$`

8. An ordered set of steps that describes the solution to a given problem in finite time is known as a(n):

   (A) syntax
   (B) semantics
   (C) flowchart
   (D) pseudocode
   (E) algorithm

9. An argument is:

   (A) a variable without a data type
   (B) ~~when the compiler disagrees~~ with your source code
   (C) the data passed to a function
   (D) a value with an undetermined point
   (E) the assignment of an incorrect data type to a variable

10. The user can interactively terminate a program data entry loop through the use of a(n)

   (A) semicolon
   (B) scope resolution operator
   (C) sentinel
   (D) break statement
   (E) pass-by-reference parameter

11. After the following statements have executed, what is the value of **dSize**?

```
double dSize = 3.9146;
dSize = (int) dSize;
```

   (A) 3
   (B) 3.0
   (C) 3.9146
   (D) 4
   (E) 4.0

12. Which of the following is the proper heading for a function that has an integer parameter and returns a value of type double?

(A) `int foo(x)`
(B) `foo(int x, double y)`
(C) `void foo(int x)`
(D) `double foo(int x)`
(E) `int foo (double x)`

13. What is true about `x >= 0` in the following line of code?

```
while (x >= 0) (cout<< "Enter new number: "; cin >>x;)
```

(A) `x >= 0` is an expression
(B) `x >= 0` is an argument to a function
(C) `x >= 0` is a compound statement
(D) `x >= 0` is an operator
(E) `x >= 0` is a variable declaration

14. Suppose you want to write a function that rotates (B=A, C=B, A=C) the contents of 3 floating point variables A, B, C. What would be a good header for this function?

(A) `int iRotate (float A, float B, float C)`
(B) `iRotate (A, B, C)`
(C) `int iRotate (&A, &B, &C)`
(D) `float iRotate(A, float &B, float &C)`
(E) `void iRotate (float &A, float &B, float &C)`

15. How many stars will the following fragment output?

```
int i;
int j;

for (i = 0; i < 12; i = i+4)
    for (j = 0; j <= 5; j = j+1)
        cout << "*";
```

0 4 8 ~~12~~ 3 times

6x3 = 18

6 times

(A) 5
(B) 15
(C) 18
(D) 60
(E) 66

16. What is the order in which the following functions are called? Assume that the necessary function prototypes/declarations appear above function main.

```
void main (void) {
  john();
}

void john (void) {
  ringo();
  paul();
}

void paul (void) {
  ringo();
}

void george (void) {
  //* here comes the sun */
}

void ringo (void) {
  george();
}
```

Main – john – ringo – george
→ paul – ringo – george

(A) john, ringo, george, paul, ringo, george
(B) main, john, ringo, paul
(C) main, john, paul, george, ringo
(D) john, paul, george, ringo, main
(E) main, john, ringo, george, paul, ringo, george

17. What arithmetic expression is implemented by the following recursive function?

```
int FunctionRecurse(int X, int Y) {
   if (Y == 0)
      return 1;
   else
      return X * FunctionRecurse(X, Y-1);
} // end fn. FunctionRecurse
```

(A) X * Y!
(B) X^N * Y!
(C) X* Y^N
(D) X! * Y-1
(E) X^Y

III. Analyzing & Writing Code: TO BE ANSWERED ON THIS QUESTION SHEET.

18. (3 marks) What is the value of x for the given values of i *after* this code fragment has executed? Assume that the code fragment is executed anew for each value of i.

```
X = 10;
if ( i > 5 )
{  if ( i > 10 )
      x = i * 10;
   else
      x = i * 5;
}
```

Indicate the value of variable x in the space below for each specific value of variable i.

	Initial Value of i	Value of x	
i)	i = 7	25	7×5
ii)	i = 3	10	hasn't changed
iii)	i = 14	140	14×10

(3)

19. (8 marks) The following C++ program is supposed to allow a user to enter in a list of years (integer values, terminating list by a negative number) and should identify all of the *leap years* entered as well keep a count of the total number of leap years entered by the user. Note that a leap year is a year that is evenly divisibly by 400 OR a year that is evenly divisible by 4 but not evenly divisible by 100. Fill in the blanks to complete the program.

```
#include <iostream.h>
int LeapYear(int);
main ()
{
   int iYear, _iCount_______;
   cout << "Enter a year to determine if the year is a leap year (or
           enter a negative number to quit): ";
   cin >> iYear;
   while ( _iYear ≥ 0____________ )
   {
      if (LeapYear(_iYear_____) == ___1___)
      {
         cout << "   " Is a leap year " " << endl;
         _iCount++.________; //incremented by one
      } // end if stmt.
      cout << "Enter another year (or a negative number to quit): ";
      cin >> iYear;
   } // end while loop
   cout << "\nThe total number of leap years entered is: "
        << iCount << endl;
} // end fn. Main


// Function LeapYear returns 1 if a given year is a leap year and
// returns 0 if a given year is not a leap year
int LeapYear ( _int iYear_______ )
{
   if ( ((iYear % 400) == 0) ||
      ( _(iYear % 4 == 0) && (!(iYear % 100 == 0))_ ))
         return 1;
   else
      _return 0_______;
} // end fn. LeapYear
```

(7)

20. (7 marks)

a) A number is said to be *prime* if it has no divisors other than 1 and the number itself. Construct a C++ function named **Prime** that returns 1 if a given number is prime and 0 if it is not a prime number.

```
int Prime(int iNumber)
{   int iCount;
    int iFlag=0;   // initialize
    if (iNumber < 0)
            iNumber = iNumber * -1;   // take the absolute value

    for (iCount = 2; iCount < iNumber ; iCount++)
    { if  (iNumber % iCount == 0)
            iFlag = 1;    // set flag to 1 because it found a factor
    }  //end for loop.
    if (iFlag == 1)
            return 0;    // if not a prime number return 0
    else
            return 1;    // if a prime number return 1
}  //end Prime
```

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b) Illustrate how you would call/invoke and use the function **Prime** that you defined in part (a). Provide a short C++ code segment to answer this question.

```
#include <iostream.h>
void main()
{ int iTestNumber;
  cout << "Enter an integer: ";
  cin >> iTestNumber;
if (Prime(iTestNumber) == 1)     //call the function Prime with arg iTestNumber.
  cout << "The number is prime ";
else
  cout << "The number is not prime";
}
```

2

/* ~~~~~~~~~~~~~~~~ ~ The End ~ ~~~~~~~~~~~~~~~~ */

7

CMPT123

Student Name:_ ___Student Number

1. Short Snappers (12 Marks)

a) A program that translates a high-level language into machine language is called a compiler.
b) True or False. It is faster to solve a problem by going directly to the coding phase for the program of the problem. *Ans*: False
c) A function should use pass by value if the programmer wants data to flow (*circle the correct answer*):
   i) Only out of the function
   ii) Only into the function
   iii) Both into and out of the function
   iv) Pass by reference is dangerous and should not be used.

(d-i) True or False: The following are characteristics that a file and a variable have in common.

d) Information can be retrieved from both *Ans*: T
e) Both are known outside the program *Ans*: F
f) Information can be stored in both *Ans*: T
g) Can exist before, after, as well as during the program execution *Ans*: F
h) Large quantities of information can be stored in either *Ans*: F unless the machine has LOTS of memory. Also depends on definition of Large.
i) Neither is know before or after a program executes, is known only during execution. *Ans*: F

(j-l) (Legal or illegal) Which of the following are legal access to the class or struct members?

```
struct S            class C
{                   {
    int x;          public:
    int y;              int x;
};                      int y;
                    private:
                        int z;
                    };
S s;                C c;
```

j) s.x *Ans*: Legal
k) c.x *Ans*: Legal
l) c.z *Ans*: Illegal

Student Name:_ ___ _Student Number:_ __

2. (2 marks) Here is a program. It compiles without complaint, but this program is in error. Find and explain any errors.

```
#include <iostream.h>
int main()
{
    int x;  ← (1)
    x=x+3;
    cout<<x<<endl;
    return 0;
}
```

← probably be safer to include parenthesis

(1) X's values hasn't been initialized... will use whatever was in that memory location prev.

3. (4 marks) Write Boolean expressions that represent the given English expressions. ~~Answers~~

a) alpha is greater than 1
   if (alpha > 1)
   ~~return (true)~~;
b) x is odd
   if (x%2 != 0)
   ~~return (true)~~;
c) x and y are odd
   ~~if (x%2 != 0, y%2 != 0~~
   if ((x%2 != 0) && (y%2 != 0))
d) ch is an upper case alphabetic character (between 'A' and 'Z')
   ~~if (ch isupper(ch))~~
   returns a boolean value
   (ch >= 'A') && (ch <= 'Z')

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4. (4 marks) Here is a collection of if-else statements with semicolons in various places. Assume all variables have been declared and initialized. Which of those are correct and are likely to give the programmer's intent? What are correct but unlikely to give the programmer's intent? Give the error for the remaining.

a)
```
if(a>b);
    a=b;
    else
    b=a;
```
→ wrong!

← the else will give error.

Ans: The semicolon will make the program think that if is an empty if statement... not ~~the~~ intention... however, it will still ~~be~~ work (just the wrong way) ~~a = b~~ ; ~~is~~ ~~initialized~~ ~~will~~ ~~run~~ ~~every~~ ~~time~~. a will always equal b.

b)
```
if(a>b)
    a=b;
    else;
    b=a;
```
→ wrong

Ans: This semicolon ~~is~~ ~~an~~ will assume an empty ~~else~~ statement. Though it'll work, it will always set b = a.

c)
```
if(a>b)
    a=b;
    else
    b=a;
```

Ans: Correct... will give desired results.

d)
```
if(a>b)
    a=b
    else
    b=a;
```
→ missing semicolon

Ans: Syntax error. Won't compile.

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5. (3 marks) Consider the following function and code segment.

```
void eleven( int first, int& second)
{
      first = 17;
      second = first + 1;
}
int main()
{
      // other code …
      int j = 4;
      int k = 3;
      eleven(j, k);
      // other code …
```

After the call to `eleven(j,k)`, what is the value of j and k? Why?
(*Note: The allocated space does not imply that your answer requires that much space for this question.*)

j = 4

k = 17+1 = 18

int& second is pass by reference... it directly affects the memory location of the value passed through second...
∴ k is altered.
j is not changed, as it is simply pass by value. the value of j is passed, not the variable.

Student Name: ____ Student Number: ____

6. (2 marks) a) What are the differences, if any, between call by array parameter and call by reference in the context of parameter passing ?

~~Parameters gives the array's values, but, unlike call by value, the array can be modified~~

~~Reference - gives memory location of array~~
Parameter - passes array location / type
Reference - passes all info.

(3 marks) b) Write a well-documented C++ function which searches an integer array and returns the number of occurrences of a particular given value.

```
int search_array (a[], int size, int val)
/* a[] is array being search, size is the size of the array, and
val is the value being seeked */
{
int count (0); // keeps track of occurences of val
for (int n=0; n<(size); n++) // goes through all array elements
{
   if (val == a[n]) // If the value is found at this index...
   count++;        // add one to the # of occurences.
}
return (count);
}
```

Ace Spade
Ten of Hearts
Jack Spade
Queen of Hearts
5 of Hearts

JS TH AS QH SH | JS TH QH AS SH | JS SH QH AS TH | JS SH QH AS TH | JS SH TH AS QH | JS AS TH SH QH | JS AS SH TH QH

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7. (3 marks) In an application, it is required to have a function that will determine whether or not the two characters a user has entered represent a two digit number. Write an implementation of such a function which returns true if the two characters is a number and false otherwise. For example, it returns true if the two characters are 23 and false if they are 2b. The characters are supplied to the function and are not requested from the user inside the function.
Note: You are not allowed to use any other functions (e.g. isdigit) in your implementation.

```
bool two_dig (char dig1, char dig2)
{
  int val1, val2;
  val1 = int(dig1) - int('0');
  val2 = int(dig2) - int('0');
  // The above get integer values, even if an integer isn't passed.
  // Now, we look + see if the values fall in the single int. range
  if ((val1 >= 0 && val1 <10) && (val2 >= 0 && val2 <10)
     return (true);
  else
     return (false);
}
```

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8. (2 marks) a) What is an ADT?

a class which is like a black box... the user of an ADT knows what it is and does, but not how it is implemented

Eg we know how to add two ~~A~~ values that are integers (ADT), but don't know how it works.

— A collection of data w/ member functions.

-1

(3 marks) b) What are the advantages of using ADT in software development?

- It can be used easily in other programs
- Doesn't need modification to switch programs (just include the header file & functions)
- can be adapted to any imaginable program that might need it

-3

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9. A 2×2 matrix is a mathematical entity with four entries organized as follows: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. The rule to multiply two matrices is given as follows:

$$\begin{pmatrix} a1 & b1 \\ c1 & d1 \end{pmatrix} \times \begin{pmatrix} a2 & b2 \\ c2 & d2 \end{pmatrix} = \begin{pmatrix} a1\cdot a2+b1\cdot c2 & a1\cdot b2+b1\cdot d2 \\ c1\cdot a2+d1\cdot c2 & c1\cdot b2+d1\cdot d2 \end{pmatrix}.$$

a) (3 marks) Give the class **definition** of myMatrix which supports the input and output of 2×2 matrices and the above multiplication operation. The input and output must make use of overloading the extraction and insertion operators, respectively. As well, the multiplication must be supported by overloading the * operator.

```
#include <iostream>
class myMatrix
{
public:
  myMatrix (double val1, double val2, double val3, do va
  myMatrix ( );
  friend istream& operator >>(istream& ins, myMatrix& matrix);
  friend ostream& operator << (ostream& out, myMatrix& matrix);
  friend myMatrix operator * (myMatrix& matrix1, myMatrix& matrix2);
private:
  double a1, a2, a3, a4;
};
```

// Sorry 'bout the mess.

b) (3 marks) Give the implementation which overloads the operator>>. Matrices can be input in the following format: (1.2 , 2.3, 3.4 ,4.5), where a=1.2, b=2.3, c=3.4, and d=4.5. All entries are of type `double`. You do not have to do any format checking.

```
istream& operator >>(istream& ins, myMatrix& matrix)
{
  ins >> matrix.a1 >> matrix.a2 >> matrix.a3 >> matrix.a4;
  return (ins);
}
```

?

-1

Need to handle parenthesis. Data is input:

(1.2, 2.3, 3.4, 4.5)

also need to handle commas.

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c) (3 marks) Give the implementation which overloads the operator<<. Output should be of the format: (a b; c d).

```
ostream& operator <<(ostream& out, myMatrix& matrix)
{
    out << "(" << matrix.a1 << " " << matrix.a2 << "; "
        << matrix.a3 << " " << matrix.a4 << ")";

    return(out);
}
```

d) (3 marks) Give the implementation which overloads the operator * so that two matrices can be multiplied using the above mentioned rule and the operation can be expressed in C++ statement as: A=B*C, where A, B, and C are 2×2 matrices of type myMatrix.

```
myMatrix operator *(myMatrix& matrix1, myMatrix& matrix2)
{
    myMatrix product;
    product.a1 = (matrix1.a1 * matrix2.a1) + (matrix1.a2 * matrix2.a3)
    product.a2 = (matrix1.a1 * matrix2.a2) + (matrix1.a2 * matrix2.a4),
    product.a3 = (matrix1.a3 * matrix2.a1) + (matrix1.a4 * matrix2.a
    product.a4 = (matrix1.a3 * matrix2.a2) + (matrix1.a4 * matrix2.a
    return (product);
}

// Excuse my poor choice of variable names -- believe me I
// I feel dumb.
```

The End